Big Bang phase: the hot, early universe

Block Course Introduction to Astronomy and Astrophysics

Markus Pössel

Haus der Astronomie and Max Planck Institute for Astronomy

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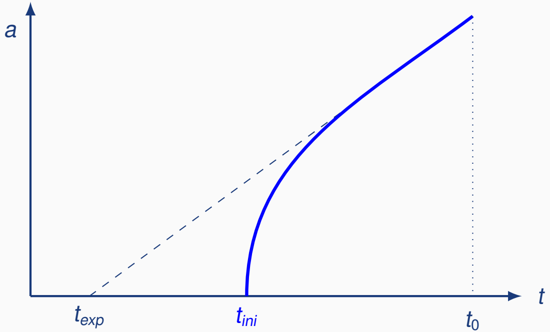
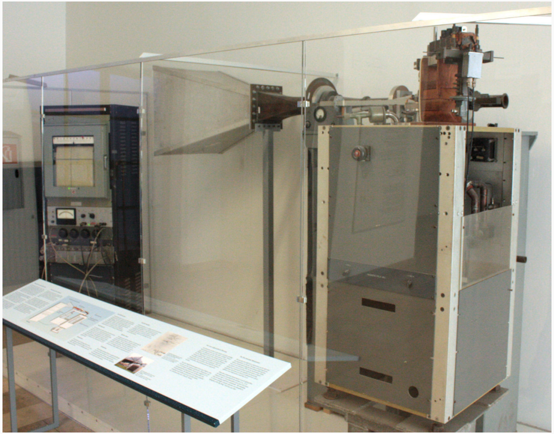


(with $t_{exp} = t_0 - 1/H_0$)

How to extrapolate to early times?

1964 Penzias and Wilson discover the Cosmic Microwave Background



Own image in Deutsches Museum, Munich

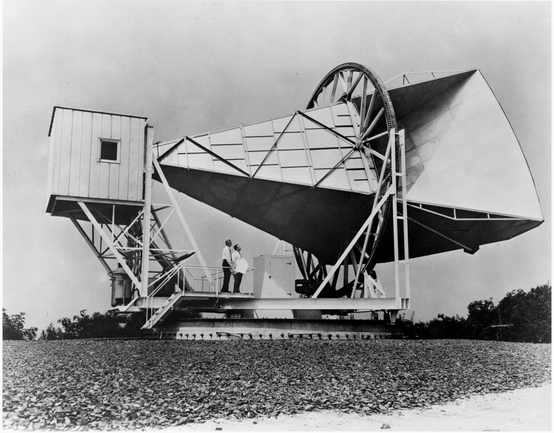
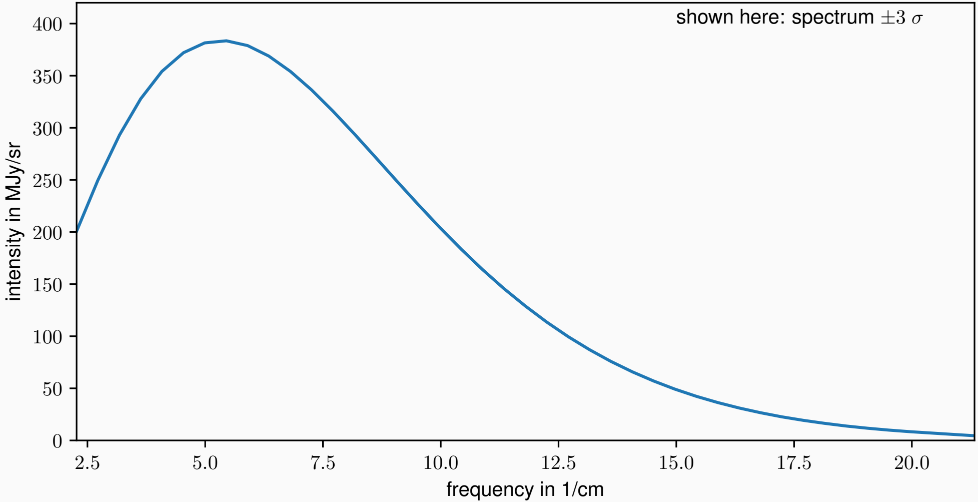


Image: NASA

Background radiation $T \sim 3.5\,\mathrm{K} \;\Rightarrow\; \Omega_R = 0.00013$

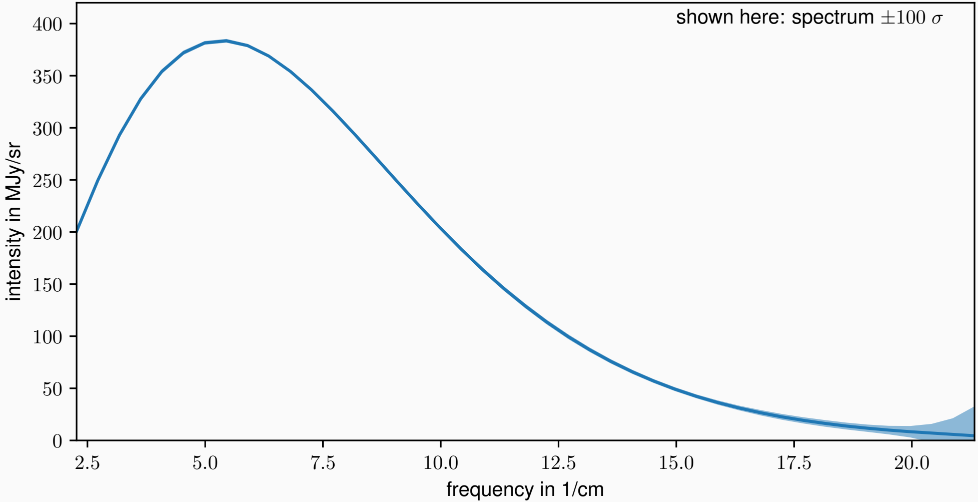
Modern value: $T = 2.7255\,\mathrm{K} \;\Rightarrow\; \Omega_R = 5 \cdot 10^{-5}$

YES, IT'S A PLANCK CURVE: COBE-FIRAS (MATHER ET AL.)



data from Fixsen et al. 1996 via http://lambda.gsfc.nasa.gov

Yes, it's a Planck curve: COBE-FIRAS (Mather et al.)



data from Fixsen et al. 1996 via http://lambda.gsfc.nasa.gov

Photon numbers (cf. today's exercise!)

From the Planck formula, by integration, photon number density is

$$n_\gamma = 4 \cdot 10^8 \frac{1}{\mathrm{m}^3}$$

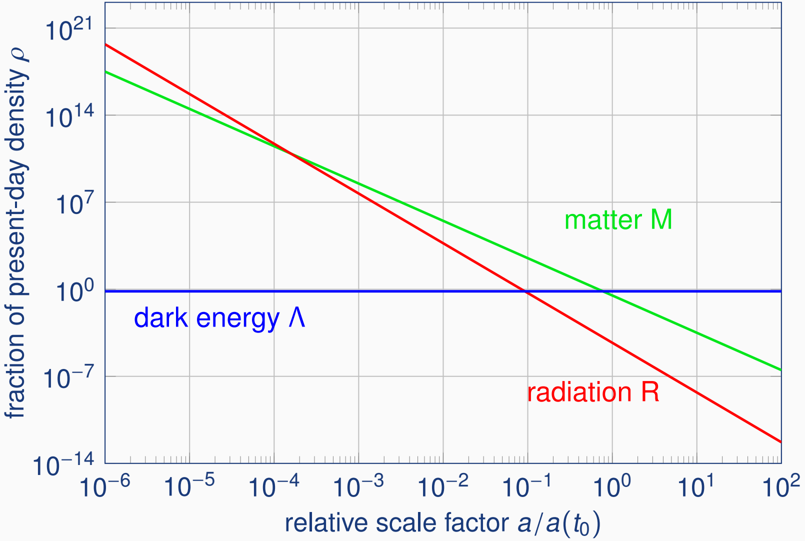
Baryonic matter is at $\Omega_b = 0.05$; if all of that were protons,

$$n_b = \Omega_b \frac{\rho_c}{m_p} = 0.27 \frac{1}{\mathrm{m}^3}$$

Baryon-to-photon ratio:

$$\eta \equiv \frac{n_b}{n_{\gamma,CMB}} = 7 \cdot 10^{-10} \sim 10^{-9}$$

Now we know there's radiation in the universe with $\Omega_R = 5 \cdot 10^{-5}$



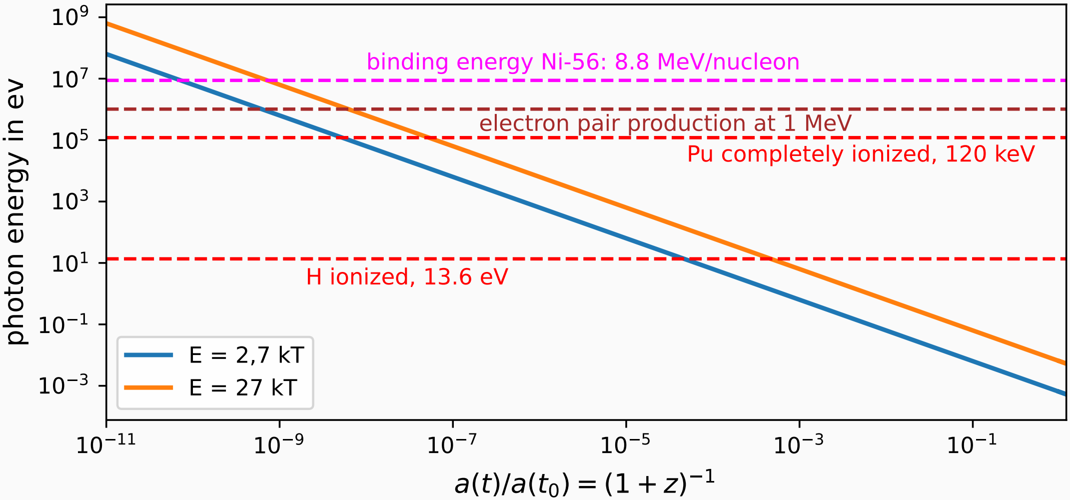
Densities scale differently:

$$\rho_M(t) \sim a(t)^{-3}$$
$$\rho_R(t) \sim a(t)^{-4}$$
$$\rho_\Lambda(t) = \rho_\Lambda$$

Radiation remains thermal, $T \sim a^{-1}$

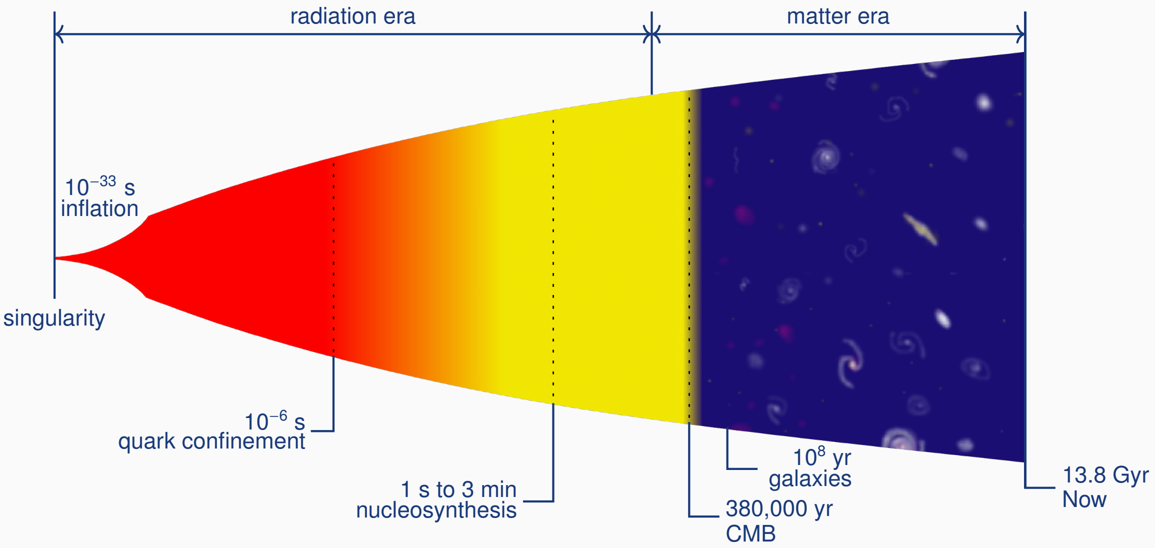
baryon-to-photon ratio remains constant:

$$\eta \equiv \frac{n_b}{n_\gamma} \sim 10^{-9} = const.$$



2.7 kT=average photon energy, 27 kT lower limit for the highest-energetic fraction $\eta = n_b/n_\gamma$

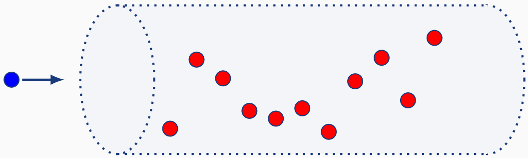
Rough sketch of cosmic history



Different phases of cosmic history

very early	quantum gravity phase	10^{-43} s	10^{19} GeV	briefly, later
	inflationary expansion	10^{-33} s		
	baryogenesis	?	10^{16} (?) GeV	lecture notes
	quark confinement	10^{-6} s	200 MeV	
early	big bang nucleosynthesis	$\sim$ minutes	10^{9} K	we'll cover the basic physics presently!
	recombination $\Rightarrow$ CMB	$4 \cdot 10^{5}$ yr	$3 \cdot 10^{3}$ K	
late	perturbations $\Rightarrow$ large-scale structure	> 1 s		next lecture
	galaxy formation	> 0.1 Gyr		later lecture

How many reactions happen to a single particle per unit time?



Particle 1 (blue) radius r_1, speed v; particless 2 (red) radius r_2
$\Rightarrow$ cross-section $\sigma = \pi\, r^2$ with $r = r_1 + r_2$

If this is like playing pool: in time interval Δt, expect $n_2\, v\, \sigma\, \Delta t$ reactions,
where n_2 is the number density for particle species 2

Many particles of type 1: collision rate density (number per unit time per unit volume)

$$C = n_1 n_2 \langle u\sigma(E)\rangle_u, \quad \text{with suitable “thermal average”}$$

with $\sigma(E)$ the (energy-dependent) cross section (usually provided by nuclear/particle physics)

The number of reactions per particle of species 1 and reaction time scale τ_C:

$$\Gamma = \frac{C}{n_1} = n_2 \langle u\sigma(E)\rangle_u \quad \Rightarrow \quad \tau_C = \frac{1}{\Gamma} = \frac{1}{n_2 \langle u\sigma(E)\rangle_u}$$

Compare this with expansion time scale

$$\tau_H \equiv \frac{1}{H(t)} = \frac{a}{\dot{a}}$$

1. $\tau_H \gg \tau_C \quad \Leftrightarrow \quad \Gamma \gg H$ for reactions that establish thermal equilibrium: local Thermal Equilibrium (LTE): adiabatic (=isentropic) change from one temperature-dependent equilibrium to the next
2. $\tau_C \gg \tau_H \quad \Leftrightarrow \quad H \gg \Gamma$: freeze-out, particle concentrations remain constant (or change because of decay, or alternative reactions); temperature decouples.

Time scale 1:

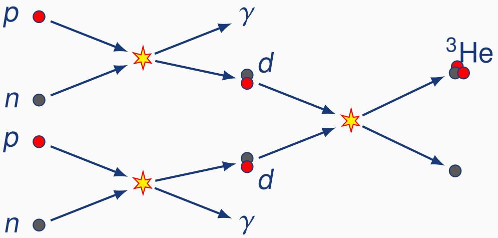
Equilibrium protons and neutrons:

$$n + \nu_e \quad \leftrightarrow \quad p + e^-$$
$$n + e^+ \quad \leftrightarrow \quad p + \bar{\nu}_e$$

Cross-section from particle physics
$\Rightarrow$ weak interactions freeze-out at

$$t \approx 1\text{s}, \quad k_B T \approx 0.7 \text{ MeV}.$$

Time scale 2:



3-particle very rare $\Rightarrow$ 2-particle reactions
$\Rightarrow$ all nucleosynthesis starts with deuterium

binding energy D is 2.2 MeV
$\Rightarrow$ begin nucleosynthesis at 27 $k_B T < 2.2$ MeV which means $T = 9.5 \cdot 10^8$ K, $t = 110$ s

Equilibrium at $t = 1$ s, $k_B T = 0.7$ MeV with $Q \equiv (m_n - m_p)c^2 = 1.293$ MeV:

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} \exp\left(-\frac{Q}{k_B T}\right) \approx \frac{1}{6}$$

From then to $t = 110$ s:
neutrons decay, with half-life $\tau_{1/2} = 614$ s:

$$\frac{n_n}{n_p} = \frac{1}{6}\left(\frac{1}{2}\right)^{110\,\mathrm{s}/614\,\mathrm{s}} \approx \frac{1}{7}$$

With 14 p and 2 n, build 1 He-4 and 12 H

$\Rightarrow$ mass ratio $Y = 0.25$

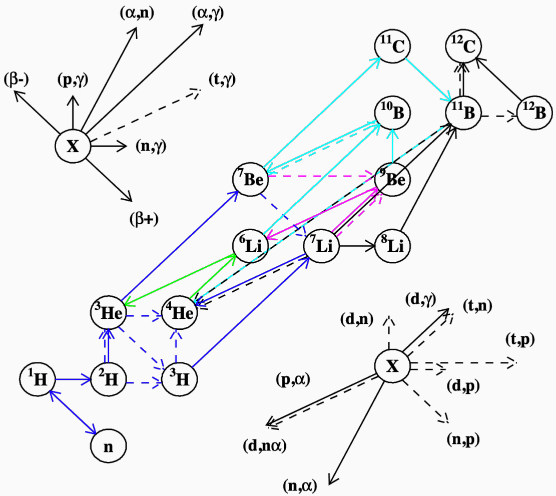
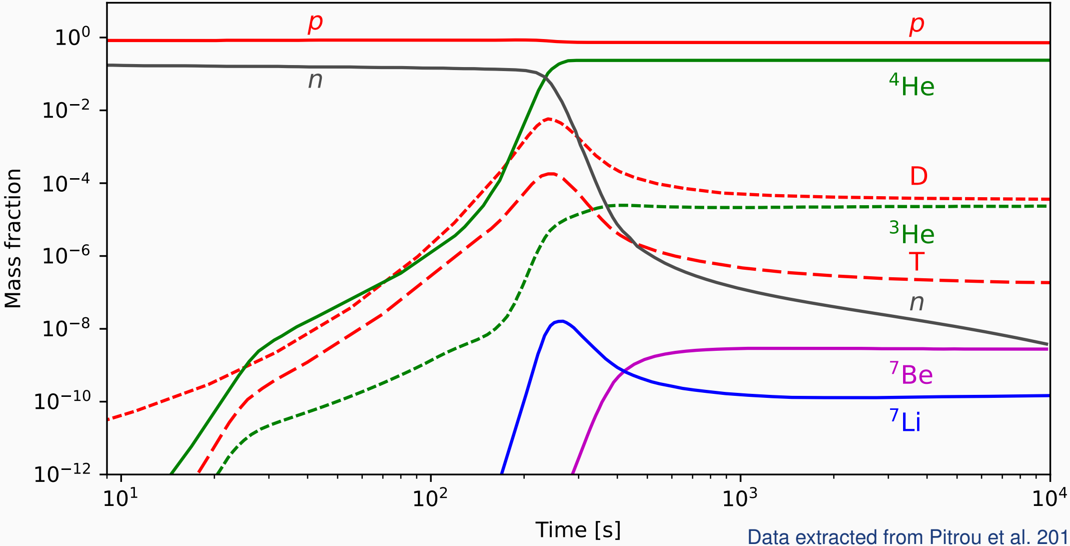


Fig. 22 in Coc 2012

Time evolution of abundances during BBN



Data extracted from Pitrou et al. 2018

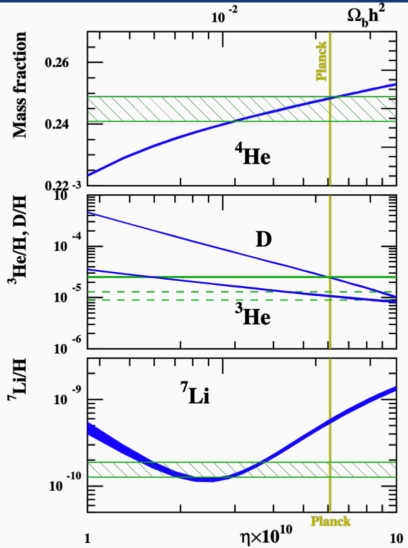


Figure 5 (minimal cosmetic alterations) in Coc et al. 2017

Going back to thermodynamics: consider equilibrium for the reaction

$$H + \gamma \leftrightarrow p + e^-$$

Non-relativistic formula for number density of particle species i in thermal equilibrium:

$$n_i = g_i \left(\frac{2\pi\, m\, k_B T}{h^2} \right)^{3/2} \exp\left(- \frac{m_i c^2 - \mu_i}{k_B T} \right),$$

with m_i mass, g_i degeneracy, μ_i chemical potential (all from Maxwell-Boltzmann)

Aspects we will not delve into, but accept:

- in equilibrium, $\mu_H = \mu_p + \mu_e$, for (blackbody) photon gas $\mu_\gamma = 0$
- $g_e = g_p = 2$ (spin $\pm 1/2$) and $g_H = 1 + 3 = 4$ (spin 0 plus spin 1 state)

Equilibrium state for ionization: Saha equation

$$\frac{n_p n_e}{n_H} = \left(\frac{2\pi m_e k_B T}{h^2}\right)^{3/2} \exp\left(-\frac{B}{k_B T}\right) \quad \text{with binding energy} \quad B = (m_p + m_e - m_H)c^2 = 13.6\ \text{eV}$$

Charge neutrality $\Rightarrow$ $n_e = n_p$, baryon number density $n_b = n_e + n_H$.

Define ionization fraction: $x_e \equiv \frac{n_e}{n_e + n_H}$,

so that

$$\frac{x_e^2}{1 - x_e} = \frac{n_e^2}{n_H(n_e + n_H)} = \frac{n_e^2}{n_H n_b} \equiv f(T, n_b)$$

Baryon number density n_b can be expressed via critical density ρ_c and because of $a(t) \sim T$ as

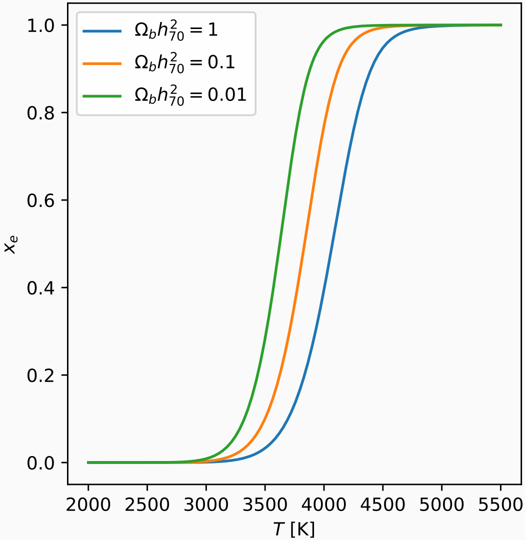
$$n_b(T) = n_{b0}\left(\frac{T}{T_0}\right)^3 = \frac{\Omega_b \rho_c}{m_p}\left(\frac{T}{T_0}\right)^3$$

Putting everything together

$$\frac{x_e^2}{1-x_e} = \frac{8\pi G m_p}{3H_0^2\Omega_b}\left(\frac{2\pi m_e k_B}{T}\right)^{3/2}\exp\left(-\frac{B}{k_B T}\right)$$
$$= \frac{9\cdot 10^{21}}{\Omega_b h_{70}^2}\left(\frac{T}{1\,\mathrm{K}}\right)^{-3/2}\exp\left(-\frac{1.6\cdot 10^5\,\mathrm{K}}{T}\right)$$
$$\equiv f(T,\Omega_b h_{70}^2)$$

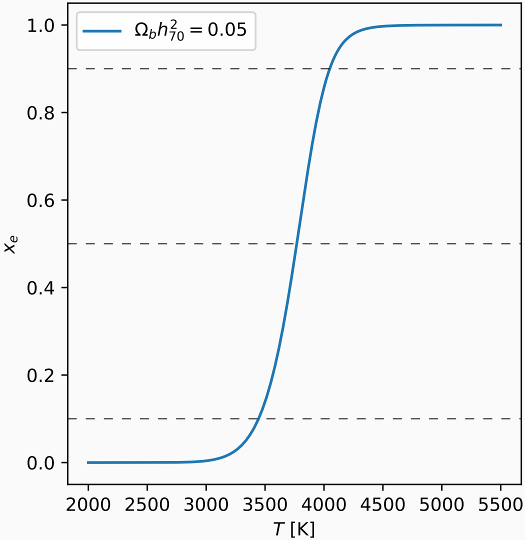
with $H_0 = h_{70}\cdot 70$ km/s/Mpc. Solution:

$$x_e = \frac{1}{2}\left[\sqrt{f[f+4]} - f)\right]$$



For the fairly realistic $\Omega_b h_{70}^3 = 0.05$:
When does recombination happen?

x_e	0.1	0.5	0.9
T	3440 K	3770 K	4050 K
z	1260	1380	1490
t	361 000 yr	316 000 yr	283 000 yr

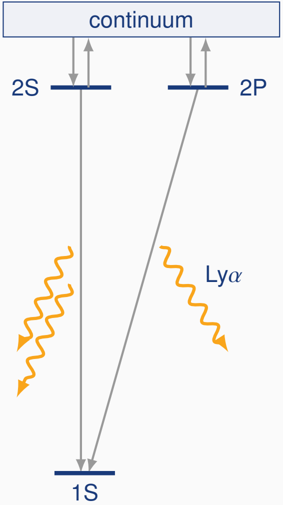


Problem with the simplification:
Photon gas treated as "heat bath" — but in reality, more photons of just the right energy to re-ionize (before they get redshifted out of resonance)

More detailed calculation:

- consider excited states in equilibrium with each other and with continuum
- rate equation for Lyα and for 2-γ-process 2S $\rightarrow$ 1S
- cosmological redshift included

$$\Rightarrow \quad z_{rec} = 1100, \quad T_{rec} = 3000\ \mathrm{K}$$



Now, for the "in brief"

What we have left out in our analysis so far (see lecture notes):

- neutrino background
- pair production $\Leftrightarrow$ matter behaves like radiation
- baryogenesis
- quark confinement

What we will now briefly address:

- quantum gravity
- inflation phase

Heuristic derivation of Planck scale:
What limits particle localization Δx?

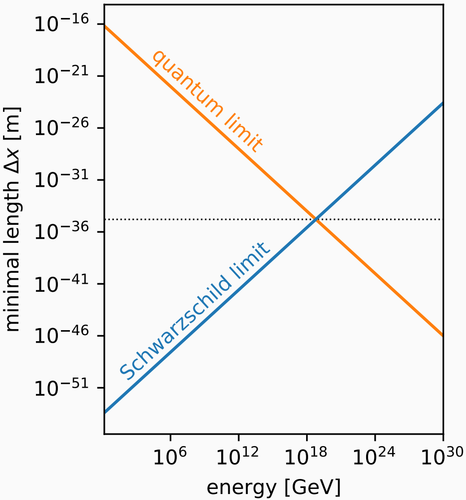
- Wave function:

$$\Delta x \sim \lambda = \frac{hc}{E} = 1.24 \cdot 10^{-15} \left(\frac{E}{1\ \mathrm{GeV}}\right)^{-1}\ \mathrm{m}$$

- Schwarzschild radius:

$$\Delta x = 2\frac{2GM}{c^2} = \frac{4GE}{c^4} = 5.3{\cdot}10^{-54} \left(\frac{E}{1\ \mathrm{GeV}}\right)\ \mathrm{m}$$

$$\ell_{pl} = \sqrt{\frac{\hbar G}{c^3}} = 1.62{\cdot}10^{-35}\ \mathrm{m}, \quad E_{pl} = 1.22{\cdot}10^{19}\ \mathrm{GeV}$$



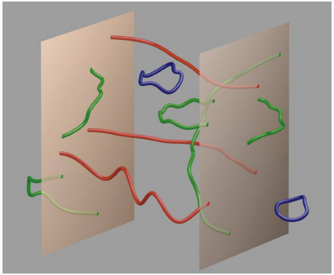
Current CMB photons: $k_B T = 0.25$ meV
$\Rightarrow$ Planck energy reached at $z = 5 \cdot 10^{22}$

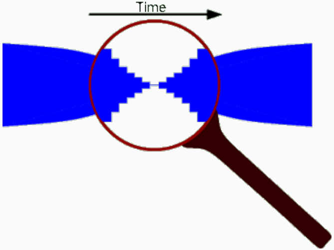
Very, very early description requires *quantum gravity theory*
(but we don't have a complete and consistent one. . .)

Candidate theories:

- String theory
- Loop quantum gravity

Both fundamentally inaccessible for experiments,
possibly predictions for CMB inhomogeneity patterns





Early phase of exponential expansion, meant to solve:

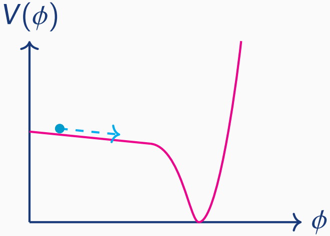
- Flatness problem
- Horizon problem
- Relic particle/monopole problem

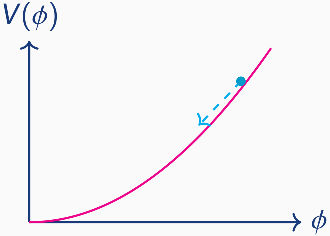
Many different flavours/models:

- old/new inflation
- chaotic inflation

with current matter content produced
via friction as inflation ends

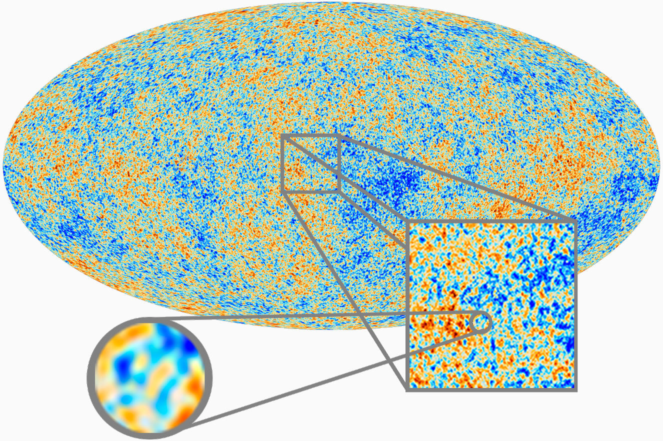
…some models have by now ruled out through
the study of CMB inhomogeneities





Particle horizon in non-inflationary models too small!

- In the night sky (transversal) particle horizon for CMB is around 3°
- Lots of causally disconnected regions in the observed CMB!
- If we assume plasma had to find thermal equilibrium since Big Bang to produce CMB, we have a problem
- Inflationary phase
  → larger particle horizon
  → problem solved



Base image: ESA Planck

Introduce time-dependent critical density and time-dependent $\Omega(t)$:

$$\rho_c(t) = \frac{3H(t)^2}{8\pi G} \quad \text{and} \quad \rho(t) = \Omega(t)\rho_c(t).$$

First-order Friedmann equation:

$$\Omega(t) - 1 = \frac{kc^2}{R_0^2(aH)^2}.$$

Specialize to absolute values, so that in a universe that is not identically flat,

$$|\Omega(t) - 1| = \frac{c^2}{R_0^2(aH)^2}.$$

Flatness problem and inflation 2

Evolution of $|\Omega(t) - 1|$ in simple universe models:

Radiation only

$$|\Omega(t) - 1| \sim \frac{1}{(aH)^2} \sim t$$

Matter only

$$|\Omega(t) - 1| \sim \frac{1}{(aH)^2} \sim t^{2/3}$$

Exponential expansion

$$|\Omega(t) - 1| \sim \frac{1}{(aH)^2} \sim \exp(-2H_0 t)$$

- Are you worried about fine-tuning? (Not everyone is.)
- Unless $k = 0$ identically, $|\Omega(t) - 1|$ must initially have been *very* small!
- Solution: initial exponential phase naturally brings $|\Omega(t) - 1|$ far down.

Generic prediction Grand Unified Theories (GUTs):
Stable particles (notably magnetic monopoles),
$m \sim 10^{16}$ GeV, sufficient numbers for $\Omega \gg 1$.

Why are there so few of them, then?

Upper limits rather low:

$$\frac{n_{monopole}}{n_b} < 10^{-29}.$$

Inflation after GUT phase: thins relic particles out

MACRO detector collaboration (Gran Sasso Laboratory), 1998–2000

Particularly interesting: inflation models make fairly generic predictions for CMB, density perturbations:

- post-inflation T: below Planck scale, $< 10^{19}$ GeV
- super-horizon fluctuations
- quantum fluctuations: adiabatic, not isocurvature perturbations
- density fluctuations power spectrum $P(k) \sim k^n$, with power-law index n close to 1
- primordial waves $\Rightarrow$ B-modes in CMB polarisation

$\Rightarrow$ more about (some of) this in the next lecture

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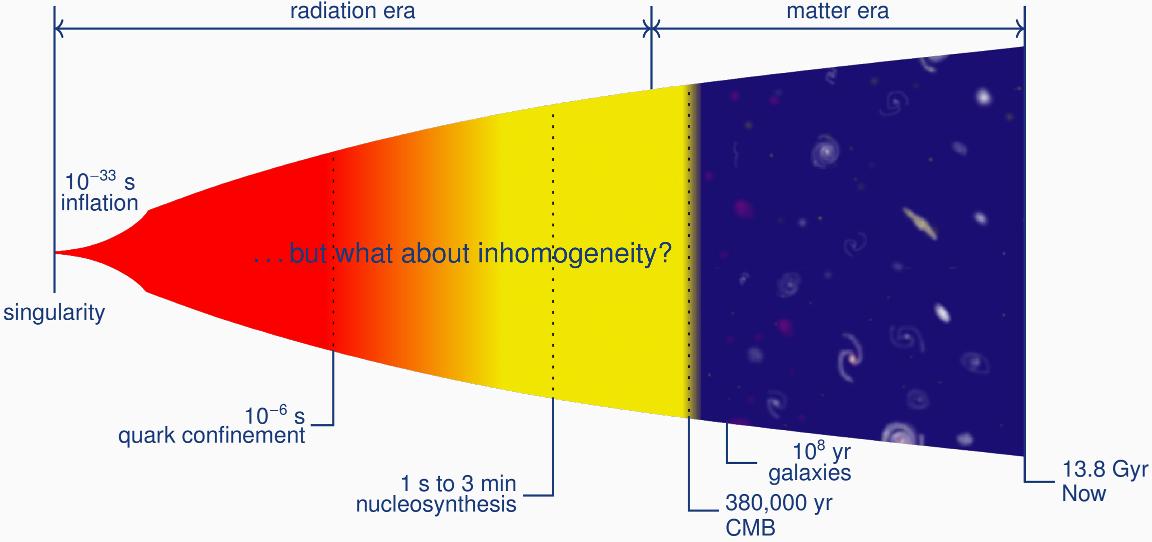
Summing up


radiation era
matter era
10^{-33} s
inflation
singularity
. . . but what about inhomogeneity?
10^{-6} s
quark confinement
1 s to 3 min
nucleosynthesis
380,000 yr
CMB
10^{8} yr
galaxies
13.8 Gyr
Now