Cosmic Distances
The Cosmic Distance Scale

- Hubble’s Law
- Type Ia Supernovae
- Cepheids
- Main-Sequence Fitting
- Parallax
- Radar Ranging

It’s the set of methods used to determine the distances to objects in the Universe.
It is based on the measurement of the round-trip time ("t") of a powerful radio signal (with speed "v") that is reflected on the atmosphere of Venus.

\[ D_{E-V} = v \left( \frac{t}{2} \right) = 42 \times 10^6 \text{ km (average)} \]

Using this method we can obtain the value of Astronomical Unit.
The maximum apparent distance of Venus from the Sun (as seen from Earth) varies between 45° and 48°. For simplicity, we take an average value between these: 46°.

\[
\frac{D_{S-V}}{D_{S-E}} = \sin 46° = 0.72
\]

\[
D_{S-V} = 0.72 D_{S-E}
\]

Since \(D_{E-V}\) is known, so it's possible to determine the distance Earth - Sun trigonometrically.

Astronomical Unit is the average Earth-Sun distance.

\[
D_{S-E} = 1 \text{ AU} \quad D_{S-V} = 0.72 \text{ AU} \quad D_{E-V} = 0.28 \text{ AU}
\]

\[
1 \text{ AU} = D_{E-V} / 0.28 = 1.5 \times 10^8 \text{ km}
\]
Parallax

When we want to determine the weight with a scale, we may note that the needle varies position each time we move our head: if we move it to the right, we will project the needle to the left, and to the left, we will see the needle projected to the right.

The shift of the object is only apparent and it is caused by a change of the position of the observer. This effect is called PARALLAX.
Another example

When you are at position B, the tree appears to be in front of this mountain.

When you are at position A, the tree appears to be in front of this mountain.

Position A  Position B
The stars have a parallax effect, annual parallax, caused by the motion of the Earth around the Sun.

The Earth revolves around the Sun drawing an ellipse whose major axis is about 300 million km.

The position, from which we observe the sky, changes over the course of the year.

Let us fix the position of a star and let repeat the observation from the same place on Earth after six months when the Earth is in the opposite point in the orbit around the Sun, we will note a small change in the star's position.
In the course of a year, a star describes a small ellipse in the sky.

The parallax method is a method similar to the triangulation. In fact, if we join the point $E_1$ with $S_1$ and $S_2$ with $E_2$, we define a triangle, which the major axis as base and the distances $S-E_1$, $S-E_2$ as sides.

By convention, the angle of the annual parallax ($p$) is equal to half the apparent angular shift of a star in 1 year, and corresponds to the vertex of the triangle which is based on the semi-axis of Earth's orbit around the Sun $(R = 150000000 \text{ km})$ and has vertex the star.
The parallax of a star decreases with distance and it has always very small values.

The nearest stars to Earth have a value of the parallax a little less than 1 second of arc.

As the distance increases, the measurement of the parallax angle becomes more difficult, as the annual change in position decreases significantly. For this reason the more distant stars maintain a fixed position and do not exhibit apparent shifts.
To derive the distance to a star directly from the value of the parallax, it is convenient to adopt a particular unit of measurement: parsec (pc).

1 parsec is the distance at which a body subtend a parallactic angle of 1” of arc.

If we express the distance in parsecs and assign the value 1 to the distance Sun-Earth = “R”, “d” is \( d = \frac{1}{p} \)

To obtain it we use trigometric ratios:
\[
d = \frac{R}{\sin p} = \frac{1}{\sin p}
\]

Since the angles with which we deal are very small, we use the approximation
\[
\sin p \approx p
\]

\[ d (\text{pc}) = \frac{1}{p} \]
By studying the annual shifts in the positions of the stars, we are able to measure parallaxes corresponding to a maximum distance of 50 pc.

The measurement of parallax seems very easy, but in reality it is necessary to use tools that reach accuracies much greater than those of a "normal" telescope, that does not reach the required accuracy.

With the Spacecraft Hypparcos we can measure parallaxes up to 300 pc.
By the end of 2013 the Spacecraft Gaia will be launched and it will be able to measure parallaxes up to 10,000 parsec.
Proxima Centauri is the nearest star to the Solar System. It has a parallax of 0.775\,"; so its distance is 
\[ \frac{1}{0.775} = 1.295 \text{ pc} \]

Sirius is the brightest star in the night sky. It has a parallax of 0.380\,"; so its distance is 
\[ \frac{1}{0.380} = 2.632 \text{ parsec} \]
Astronomical Unit (au): average Earth-Sun distance (149,600,000 km)

Parsec (pc): the distance from the Sun to an astronomical object which has a parallax angle of one arcsecond (1/3,600 of a degree).

Light Year (l.y.): the distance that light travels in a vacuum in one year with a speed of ~ 300,000 km/s

Conversion Factors

- $1\text{ au} \approx 4.85 \times 10^6\text{ pc} \approx 15.81 \times 10^6\text{ ly}$
- $1\text{ light-year} \approx 63.24 \times 10^3\text{ au} \approx 0.31\text{ pc}$
- $1\text{ parsec} \approx 20.62 \times 10^4\text{ au} \approx 3.26\text{ ly}$
CAUTION!

LIGHT YEAR is NOT a unit of measurement of time.

It is correct to say that the image of a celestial body, which is far a certain number of light years, shows us that celestial body as it was the same number of years ago, and not at this time.

The distance Earth - Andromeda is 2 million ly, so if an observer could see the Earth from Andromeda, he would see the Earth today as it was 2 million years ago.
Main – Sequence Fitting

Both the temperature and the brightness of a star can be derived from the analysis of its spectrum.

The knowledge of these parameters allows us to locate a star in the HR diagram and then to derive the absolute magnitude ($M$).

Knowing $M$ and the apparent magnitude ($m$) we can obtain the value of the distance $d$:

$$M - m = 5 - 5 \log d$$

This method is called spectroscopic parallax.
The determination of spectroscopic parallaxes is particularly useful to determine the distance of the stars that belong to a cluster.

The star clusters are generally far away, so the direct determination of the distance of their individual stars is very uncertain.

Almost all clusters have a small angular diameter; therefore they are aggregates of stars of modest size compared to the great distance that separates them from us.

Then we can assume that the member stars are at the same distance from Earth.
Using the apparent magnitudes instead of absolute ones, we can build the HR diagram of the stars in the cluster M67. The main sequence of the unknown cluster, thus obtained, differs from that of a known cluster for the different scale on the y-axis.

From the translation that must be introduced in order to obtain the overlap of the two sequences, we can determine the value of the average distance of the stars of the cluster from Earth.
Standard Candle

It's an object, such as a star, that lies within a galaxy and for which we know the luminosity.
Comparing the known luminosity with the observed luminosity (apparent magnitude), we can calculate the distance \(d\): 

\[ M - m = 5 - 5 \log d \]

To be useful, standard candles should have the following properties:

- **They should be luminous**, so we can see them out to great distances.
- **We should be fairly certain about their luminosities**.
- **They should be easily identifiable**.
- **They should be relatively common**, so we can use them to determine the distances to many different galaxies.

**Type of Standard Candles:**

- **Cepheids**
- **Type Ia Supernovae**

### Spiral Galaxy M-100

Supernovae 2006X
Cepheids are variable stars; stars showing variations of luminosity in the course of their existence.

If you analyze the spectrum of a star, you can determine its temperature and its color. From these parameters we derive the luminosity.

The period of pulsation (P) is related to the luminosity of the star. The longer is the period, the higher is the luminosity.

A Cepheid with a three-day period has a brightness equal to 800 times that of the Sun; a Cepheid with a period of thirty days is 10000 times more luminous than the Sun.

Cepheid Variable Stars in Spiral Galaxy NGC3021 (Hubble Space Telescope)
Cepheids in particular are very important because, due to their high luminosity, they can be easily observed in nearby galaxies.

Comparing $M$ with the apparent magnitude $m$, we can determine the distance $d$: $M - m = 5 - 5\log d$

Through the period – luminosity relation one can determine the absolute magnitude of a Cepheid: $(M = -1.43 - 2.81 \log P)$

Owing to the great accuracy of the measurement of the pulsation and of the period-luminosity relation, Cepheids can then be used as standard candles to determine the distance of globular clusters and the galaxies in which they are contained.

Cepheids in particular are very important because, due to their high luminosity, they can be easily observed in nearby galaxies.
More recently, the Hubble Space Telescope was able to identify some Cepheid variables in the Virgo cluster, at a distance of ~20 Mpc.

In 1912, Henrietta Leavitt discovered that the periods of Cepheides variable stars in the Small Magellanic Cloud are related to their luminosity.

With Cepheid variables we can estimate the distance of the nearest galaxies up to 30 Mpc. Therefore, they are the main step connecting galactic and extragalactic distances.
To experience the emotion of the scientific research and of the astronomy, we can use a free tool available on the Internet:

EuroVo → a project developed within the European Virtual Observatory which has as goal the diffusion of data and software EuroVo to the public.

This project provides examples, teaching modules and simplified professional software.

To download it: http://wwwas.oats.inaf.it/aidawp5/eng_download.html?fsize=medium
Worth of mention is Aladin that has several built-in functions that allow the image handling and a quick analysis.

We can see two possible applications of Aladin on cosmic distances:

**Distance of Barnard Star**

**Star Aladin**
The distance of the Barnard star from Earth is:

\[ d = \frac{1}{0.54831} \approx 1.82 \text{ pc} \]

548.31 mas (milliarcsecond) = 0.54831 as
In 1924 Edwin Hubble identified some Cepheids in the Andromeda Galaxy; he concluded that it was located far away from our Galaxy.
We choose the most recent catalog, it means the period of the stars, but not all have it shown.
To see only the stars of which the period is indicated, we create a filter.

To calculate absolute magnitude

To calculate the distances
It's the distance of all Cepheids

By averaging, we find the approximate distance to the Andromeda Galaxy: $(2.52 \pm 0.14) \times 10^6$ ly
Type Ia Supernovae

To extend the distance scale even further, we need brighter objects but just as reliable as standard candles, and the best candidates are the Type Ia Supernovae.

These are used beyond 30 Mpc and occur when a white dwarf in a close binary system accretes enough matter from its companion to blow itself apart in a thermonuclear explosion.

If a Type Ia supernova is seen in a distant galaxy and its maximum apparent brightness measured, we can find the galaxy’s distance.

The inset shows one of the 10 most distant and ancient Type Ia supernovae.
Not all Type Ia supernovae are equally luminous, but a simple relation holds: the slower the brightness decreases, the more luminous the supernova.

Using this relationship, we have measured distances to supernovae more than 1000 Mpc from Earth.

This technique can be used only for galaxies in which we happen to observe a Type Ia supernova. Now we are able to identify many dozens of these supernovae every year, so the number of galaxies whose distances can be measured in this way is continually increasing.
The figure shows the ranges of application of several important methods for the determination of astronomical distances. Because these ranges overlap, one technique can be used to calibrate another. For this reason, we go to great lengths to check the accuracy and reliability of our standard candles. A change in distance-measuring techniques used for nearby objects can also have substantial effects on the distances to remote galaxies.
As an example, the distance to the galaxy M100 shown in the figure is determined using Cepheids. A Type Ia supernova has been seen in M100, as the figure shows, and its luminosity is determined using the Cepheid-derived distance to M100. Any change in the calculated distance to M100 would change the calculated luminosity of the Type Ia supernova, and so would have an effect on all distances derived from this.

Because one measuring technique leads us to the next one like rungs on a ladder, the techniques shown in the figure of the previous slide, are referred to collectively as the distance ladder.
During the 1920s, Edwin Hubble, by observing the luminosity and pulsation periods of Cepheid variables in many galaxies, was able to measure the distance to each galaxy.

Many spectral lines emitted by chemical elements similar to those present in the Sun are found in the light emitted by distant stars.

Analyzing the spectra we can notice that the lines produced by the distant stars correspond to those emitted by known elements only with a systematic shift towards longer wavelengths.
Hubble found that most galaxies show a shift toward the red portion of the spectrum. The redshift \( z \) is found from the ratio: 
\[
z = \frac{\lambda - \lambda_o}{\lambda_o}
\]

\( \lambda \) is the wavelength of the line of the spectrum emitted by a star in motion and the corresponding wavelength \( \lambda_o \) of the same line of a reference spectrum, usually the solar one.

Hubble found a correlation between the distance to a galaxy and its redshift: nearby galaxies are moving away from us slowly, while more distant galaxies are rushing away from us much more rapidly.
The redshift is similar to a Doppler effect, but in reality its physical nature is different.

It is the basis of the most shocking cosmological theory: the expansion of the Universe. It creates new space between source and observer, "stretching" the wavelength.
The movement away from our of the other galaxies is called recession motion. The relation between the recession velocity \( v \) and distance \( d \) of a galaxy from Earth is expressed by the Hubble law: \( v = d H_0 \).

\( H_0 \) is the Hubble constant and its value is constantly revised. It is constant because if at this time we repeat its measure, again, in any other point in the universe, we would get the same value of \( H_0 \); but it varies over time.

The evolution of \( H_0 \) is due to the effects of the gravitational force of the matter in the Universe that tends to slow the expansion, and the so-called dark energy, which instead tends to speed it up.

<table>
<thead>
<tr>
<th>Measured by</th>
<th>Hubble</th>
<th>Hubble Space Telescope</th>
<th>Space Infrared Telescope Spitzer</th>
<th>Planck Spacecraft</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0 ) (km/s/Mpc)</td>
<td>500</td>
<td>73,8 ± 2,4</td>
<td>74,3 ± 2,1</td>
<td>67,15 ± 1,2</td>
</tr>
</tbody>
</table>

From these measurements continuously updated, we can conclude that the sequence of measurements of the Hubble constant is converging to a value more accurately.
CAUTION!

The use of the Hubble law has some limitations. We can not use it when:

- the distances are less than a few million light years, because the gravitational effect would occur due to the proximity.

- the distances are very large, in excess of several billion light years, for two reasons: on one hand the uncertainties increase with distance; on the other hand, the proportionality speed - distance requires knowledge of the developmental characteristics of the galaxy, to as the Hubble parameter that has characterized the expansion of the universe in the distant past (cosmological parameters were measured using the Type Ia Supernovae as standard candles).

Finally, it should be noted that this law expresses a situation existing at a given moment and not as the velocity of an object changes when it moves away.
Given the value of $H_0$, we can use the Hubble law to find the distance $d$ to the galaxy if we know how to determine the recession velocity $v$ from the laws of the Doppler effect.

If the redshift and recession velocity are not too great, we can use the following equation:

$$v = c z$$

where $c$ is the speed of light.

Not all galaxies are moving away from us! Some of the nearest galaxies, including Andromeda Galaxy, are actually approaching us and have blueshifts rather than redshifts.
Let's take an example, for us closer, to understand the orders of magnitude with which we deal in the distance scales: if the Sun was an orange, the Earth would be the head of a needle turning 15 meters away around the orange, Jupiter was a cherry, turning around 77 meters, Pluto a grain of sand at 580 meters and Proxima Centauri another orange at 4000 km!
Appendix A: Knowledge Required

*Trigonometric Ratios*

- $b = a \sin b$
- $c = b \tan g$
- $b = a \cos g$
- $c = b \cot g b$
- $c = a \sin g$
- $b = c \tan b$
- $c = a \cos b$
- $b = c \cot g g$

*Brightness and Luminosity*

Brightness is the amount of light received by an object.
Luminosity is the amount of light emitted from the star in the unit of time. Its unit of measurement is “candela”.
Magnitude

The apparent magnitude \((m)\) of a celestial body is a measure of its brightness as seen by an observer on Earth, adjusted to the value it would have in the absence of the atmosphere.

The absolute magnitude \((M)\) is the measure of a celestial object's intrinsic brightness.

\[ M - m = 5 - 5 \log d \]

The difference \((M - m)\) is called distance modulus.

HR diagram

The HR diagram collects key information on stars. Each star is identified by two coordinates:

- In the x-axis is the spectral type,
- In the y-axis the absolute brightness.

The spectral types indicate the surface temperature of the star, they are arranged in the upper x-axis in order of decreasing temperature.

The central area of the diagram, that extends diagonally from top left to bottom right, is called the main sequence. It collects the highest number of stars in the diagram.
Doppler Effect

It is a physical phenomenon which consists in the apparent change in frequency or wavelength of a wave perceived by an observer who is in motion or at rest relative to the source of the waves, also in motion or at rest.

Example

The Doppler Effect for a Moving Sound Source

If the car with siren is approaching us, we hear an higher sound than the same one issued by the source when it is stationary; when it goes away it appears lower.