The Virial Theorem and its astronomical applications

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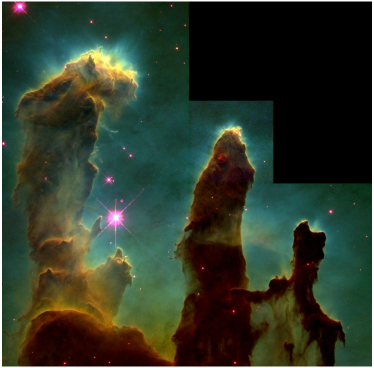
Contents

This work is divided in three parts:

FIRST SECTION:

The formation of Stars

Introduction



Eagle Nebula (source NASA)

Stellar sector ngc 2467 (source NASA)

PHILOSOPHICAL TRANSACTIONS.

I. *The Stability of a Spherical Nebula.*

By J. H. Jeans, *B.A., Fellow of Trinity College, and Isaac Newton Student in the University of Cambridge.*

Communicated by Professor G. H. Darwin, *F.R.S.*

Received June 15,—Read June 20, 1901. Revised February 28, 1902.

Introduction.

§ 1. The object of the present paper can be best explained by referring to a sentence which occurs in a paper by Professor G. H. Darwin.* This is as follows:—

"The principal question involved in the nebular hypothesis seems to be the stability of a rotating mass of gas; but, unfortunately, this has remained up to now an untouched field of mathematical research. We can only judge of probable results from the investigations which have been made concerning the stability of a rotating mass of liquid."

In so far as the two cases are parallel, the argument by analogy will, of course, be valid enough, but the compressibility of a gas makes possible in the gaseous nebula a whole series of vibrations which have no counterpart in a liquid, and no inference as to the stability of these motions can be drawn from an examination of the behaviour of a liquid. Thus, although there will be unstable vibrations in a rotating mass of gas similar to those which are known to exist in a rotating liquid, it does not at all follow that a rotating gas will become unstable, in the first place, through vibrations which have a counterpart in a rotating liquid: it is at any rate conceivable that the vibrations through which the gas first becomes unstable are vibrations in which the compressibility of the gas plays so prominent a part, that no vibration of the kind can occur in a liquid. If this is so, the conditions of the formation of planetary systems will be widely different in the two cases.

With a view to answering the questions suggested by this argument, the present paper attempts to examine in a direct manner the stability of a mass of gravitating gas, and it will be found that, on the whole, the results are not such as could have been predicted by analogy from the results in the case of a gravitating liquid. The

* "On the Mechanical Conditions of a Swarm of Meteorites, and on Theories of Cosmogony," 'Phil. Trans.,' A, vol. 180, p. 1 (1888).

Front page of the original document

This criterion is called the **Jeans mass** from Sir James Jeans (1877 – 1946)

The Virial Theorem

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and the equation of **Navier-Stokes**.

$$\frac{d\mathbf{v}}{dt} = \mathbf{a} - \frac{1}{\rho}\nabla \mathbf{P} + \omega \nabla^2 v$$

a is the acceleration due to external forces, **P** is the pressure due to the mass of fluid and the ω is the kinematic coefficient of viscosity.

The Virial Theorem

Now we suppose that the cloud has spherical simmetry, in which the viscous effects are negligible and that the body is in Hydrostatic Equilibrium.

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$$\nabla \mathbf{P} = \rho \mathbf{g} \quad and \quad \frac{1}{\rho}\frac{\partial P}{\partial r} = -\frac{GM_r}{r^2} \tag{1}$$

The Virial Theorem

Than it is possible to proof that for a cloud in Hydrostatic Equilibrium holds the next formula called the **Virial Theorem**.

$$2K + U = 0 \tag{2}$$

where K is the total thermal energy (internal kinetic energy) of the star and U expresses the gravitational potential energy.

Jeans criterion

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If now we consider a spherical cloud of constant density,the gravitational potential energy is approximately

$$U = -\frac{3}{5}\frac{G(M_c)^2}{R_c} \quad (3)$$

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where M_c and R_c are the mass and the radius of the cloud. If the interstellar cloud is approximated as being **isothermal** and constant density ρ then the thermal energy may be written

$$K = \frac{3}{2}Nk_B T \tag{4}$$

where N is the total number of particles contained in the cloud and k_B is the Boltzmann constant.

Jeans criterion

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$$\frac{3}{5}\frac{GM_c^2}{R_c} > 3\frac{k_B T M_c}{\mu m_H} \tag{6}$$

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If we suppose constant the initial mass density of the cloud ρ_0 we have

$$R_c = \left(\frac{3M_c}{4\pi\rho_0}\right)^{1/3} \tag{7}$$

Jeans criterion

Then after substitution we obtain the condition to initiate the collapse of the cloud and is know as the **Jeans criterion**

$$M > \left(\frac{5k_B T}{\mu m_H G}\right)^{\frac{3}{2}} \left(\frac{3}{4\pi\rho_0}\right)^{\frac{1}{2}} =: M_J \tag{8}$$

where Jeans mass M_J is defined.

Jeans criterion

From the expression of the radious in the previous formula we can get the so called **Jeans radius**

$$R_J = \left(\frac{15 k_B T}{4\pi G \mu m_H \rho_0} \right)^{1/2} \tag{9}$$

The free-fall timescale

Other important parameter which we have to introduced is the **free-fall timescale**.

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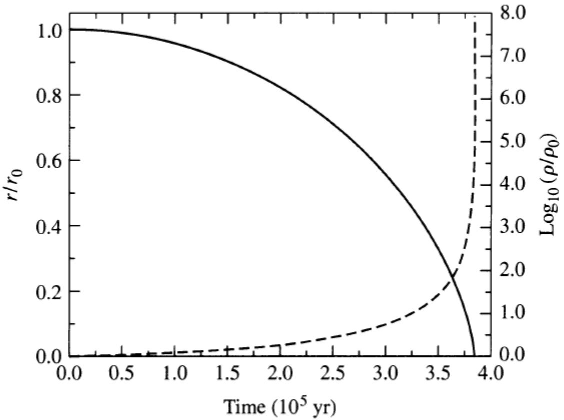
$$t_{ff} = \left(\frac{3\pi}{32} \frac{1}{G\rho_0} \right)^{1/2} \approx \frac{1}{\sqrt{G\rho_0}}$$

where ρ_0 is the initial density of the body.

Homologous Collapse

So far, the instruments present here let to an **Homologous Collapse**



Problems and observations

(i) stars frequently tend to form in group

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(ii) only a few numbers of cloud let to the formation of stars

Adiabatic evolution process

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On the other hand thanks to Stefan-Boltzmann's equation we may express the radiated luminosity as

$$L_{rad} = 4\pi R^2 e\sigma T^4$$

where we introduced the efficiency factor $0 < e < 1$, to indicate the deviation from thermodynamic equilibrium

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$$M_J = \left(\frac{4\pi}{G^{3/2}} R_J^{9/2} e\sigma T^4 \right)^{2/5}$$

Then we arrive at the estimate required

$$M_{J_{min}} = 0,03 \left(\frac{T^{1/4}}{e^{1/2}\mu^{9/4}} \right) M_\odot$$

where T is expressed in kelvin and $M_\odot$ is the mass of the sun.

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References:

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SECOND SECTION:

The Fundamental Plane and the problem of "Tilt"

Fundamental Plane of ellipticals - Introduction

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For example, with the introduction of CCD, in the second half of Seventies, the scientists were able to make more accurate photometric and kinematic measurements...

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Fundamental Plane of ellipticals - Some Parameters

There are *two fundamental sets* of elliptical galaxies basic **structural parameters**: one is the set of shape parameters and the second consists of the shape-independent parameters. We are interested in the second type

The second type parameters are, for example: the *effective radius* ($\mathbf{R_e}$), the *effective surface brightness* ($\mathbf{I_e}$), the *central velocity dispersion* (σ_0), the *luminosity in various bands X* ($\mathbf{L_X}$), the *mean color* ($\mathbf{B-V}$) and the *line-strenght indices of magnesium* ($\mathbf{Mg_2}$)

Fundamental Plane of ellipticals - Definition of some important parameters

We recall the definition of some of these parameters, (we will focus only on those that are involved in the following):

- the EFFECTIVE RADIUS (R_e) of a galaxy is the radius at which one half of the total light of the system is emitted within this radius (spherical symmetry)

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- the SURFACE BRIGHTNESS (I_e): the overall brightness of an extended astronomical object (a galaxy, a cluster, a nebula) can be measured by its *apparent magnitude*. This is a limited tool for our purposes because it is clear that, on equal apparent magnitude, an extended object will be harder to see than a star. We need an extra parameter: the surface brightness give an indication of how easily observable the object is

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- the VELOCITY DISPERSION (σ) is the statistical dispersion of velocity about the mean velocity of a group of objects

Fundamental Plane of ellipticals -

Some correlations among parameters of ellipticals

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Mg_2 - σ_0: The intensity of Magnesium increases with both the luminosity of galaxy (*Faber*, 1973) and the central velocity dispersion (*Burstein et al.* 1988; *Bernardi et al.* 2003)

Color - Magnitude: There is a correlation between the absolute magnitude an the color of galaxies; so the brighter galaxies are more red than fainter galaxies

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L - σ_0: In 1976 *Faber and Jackson* discovered the following relation:

$$L \propto \sigma_0^n$$

where $3 \leq n \leq 5$ and, the σ_0 is referred, generally, to the central region of radius $R_e/8$

The fundamental plane

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The innovation of this discover was the definition of the three dimensional space of three observable σ_0, R_e and I_e. In this space, the elliptical galaxies are **not uniformly disposed**, but are concentrated on a logarithmic plane, named ***fundamental plane***, that is a relation like:

$$log R_e = \alpha \ log \sigma_0 + \beta \ log I_e + \gamma$$

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An important property of the FP is the constant dispersion of the various involved observable: for example, the distribution of R_e around the best-fit (with fixed I_e and σ_0) has a dispersion that can change from 15% to 20%.

It is possible to prove that the characteristic dynamical time of ellipticals and their collisionless relaxation time is of the same order (Lynden-Bell 1967). In particular are both short with respect to the age of ellipticals.

$$\Downarrow$$

Highly perturbed galaxies are presumably caught in a non-stationary phase. ***Stationarity*** is a *sufficient* condition for the **validity of the Virial Theorem**, and so for ellipticals the Virial Theorem holds.

The theoretical interpretation of Fundamental Plane

So, assuming the validity of Virial Theorem, we can begin the calculation.

The scalar Virial Theorem, says that the *kinetic energy* T and the *potential energy* Ω in a system are correlated in this way:

$$2T = -\Omega$$

So it is true that:

$$\mathbf{G}\frac{\mathbf{M}}{\langle \mathbf{R} \rangle} = \left\langle \mathbf{V}^2 \right\rangle$$

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- G is the gravitational constant
- M the mass
- $\left\langle V^2 \right\rangle$ the mean quadratic velocity weighted by the mass
- $\langle R \rangle$ the characteristic gravitational radius, weighted by the mass

- If R_e is the effective radius of the elliptical galaxy, we can define a parameter k_R that satisfy:

$$\langle R_e \rangle = k_R \langle R \rangle$$

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and that *must reflects the kinematic structure of the galaxy*

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Finally, we know that $L = c_1 I_e R_e^2$ where I_e is the *mean superficial effective brightness* ($I_e = L/2\pi R_e^2$) so we insert this information in the above equation for M. The final result is:

$$R_e = (c_1 c_2^{-1}) \left(\frac{M}{L} \right)^{-1} \sigma_0^2 I_e^{-1}$$

The theoretical interpretation of Fundamental Plane

If we assume that every galaxy has the **same luminosity profile** (i.e. we assume that all galaxies are "homologous" in this sense) the parameter $\boxed{\mathbf{c_1}}$ is constant

(because $I_e = L/2\pi R_e^2$ *and* $L = c_1 I_e R_e^2$ *and if we substute...)*

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The parameter $\boxed{\mathbf{c_2}}$ indeed **depends on the mass and the velocity dispersion** in the galaxy (because $c_2 = (Gk_V k_R)^{-1}$). So, c_2 is constant if we assume that all galaxies are homologous in this sense too.

A priori, the resulting equation

$$\mathbf{R_e} = (\mathbf{c_1 c_2^{-1}}) \left(\frac{\mathbf{M}}{\mathbf{L}}\right)^{-1} \sigma_0^2 \mathbf{I_e^{-1}}$$

is not a defined locus in the space $(\sigma_0,\ R_e,\ I_e)$. For every point of this three-dimensional space, the values of c_2 and M/L can vary a lot.

$$\mathbf{R_e} = (\mathbf{c_1 c_2^{-1}}) \left(\frac{\mathbf{M}}{\mathbf{L}}\right)^{-1} \sigma_0^2 \mathbf{I_e^{-1}}$$

But, if the galaxies all belong to the same homologous family and the ratio M/L is constant from galaxy to another with the change in mass, we have that the equation below should define univocally the physical characteristic of the galaxy:

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The FP from the Virial Theorem

$$\mathbf{R_e} \propto \sigma_0^{\mathbf{A}} \mathbf{I_e^B} \qquad \text{with} \qquad \mathbf{A} = 2 \quad \mathbf{B} = -1$$

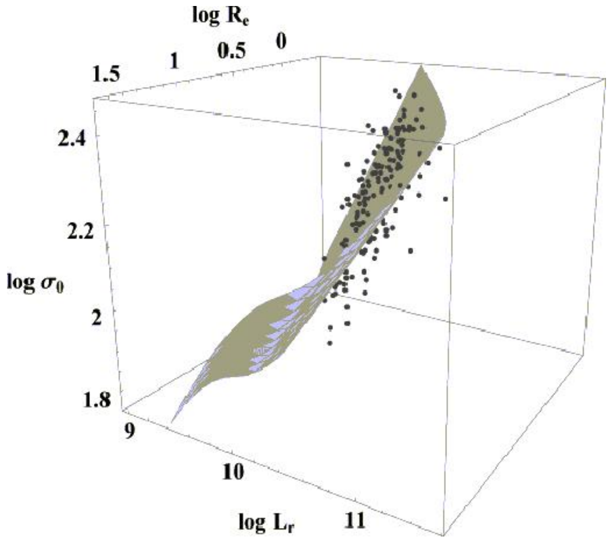
The problem of "Tilt"

The **PROBLEM**:

the experimental evidences say that the galaxies, in the $(\sigma_0, \ R_e, \ I_e)$ space, are concentrated in a plane that is considerably distant from the plane that the Virial Theorem return!

This gap between the theoretical and experimental coefficients is also named **"the tilt of fundamental plane"**

The problem of "Tilt"



The problem of "Tilt"

In the 1987 it was determined, in photometric band B, the right coefficients of the plane: $\bar{A} = 1,39 \pm 0,15$ and $\bar{B} = -0,9 \pm 0,1$.

To be more precise, when the two planes $R_e \propto \sigma_0^A I_e^B$ and $R_e \propto \sigma_0^{\bar{A}} I_e^{\bar{B}}$ are in logarithmic form, **they appear to be tilted by an angle of $\sim 15^\circ$ in all three variables**.

The problem of "Tilt"

This problem is a consequence of a wrong assumed hypothesis:

The problem of "Tilt"

This problem is a consequence of a wrong assumed hypothesis:

the ratio M/L is not constant with the change in mass of galaxy and the ellipticals are not an homologous family of galaxies in this sense

(an explanation can be the existence of Dark Matter . . .)

The problem of "Tilt"

The conclusion is that there are three important still open problems:

i) why is the FP so thin
(i.e. why is that relation so tight);

ii) why is the FP "tilted"
(i.e. why elliptical galaxies don't seem to obey the Virial Theorem);

iii) why does the FP itself exists
(i.e. what causes the properties of elliptical galaxies to be so "systematic")

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THIRD SECTION:

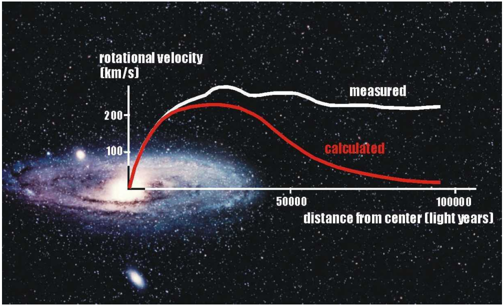
The Dark Matter

Evidence for an unseen component in spiral galaxies

- In 1970s *Vera Rubin* was studying HII regions in spiral galaxies, and she was able to plot their velocities around the galactic centre as a function of their distance from it.

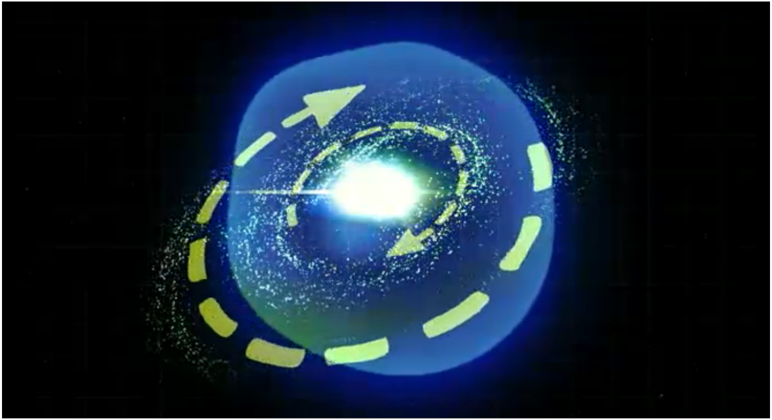
  Problem: she found that the rotational speeds of the clouds did not decrease with increasing distance from the galactic centre and, in some cases, even increased somewhat.

Evidence for an unseen component in spiral galaxies



Possible answers: even the stars in the galaxy are embedded in a large halo of **unseen matter** or Newton's law of gravity **does not hold true for large distances**.

Dark matter halo in spiral Galaxies



Why dark matter is dark

- With ordinary matter, electromagnetism affects everything from chemistry to luminosity to electric and magnetic fields and even the pressure of stellar winds; thus electromagnetism plays an important role in determining the arrangement of ordinary matter, which is often irregular.

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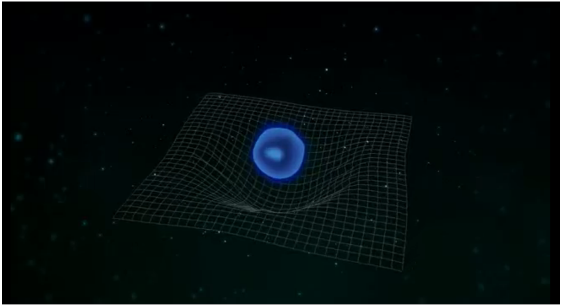
- Except through gravitation, dark matter **does not interact** (or interacts only very weakly) with itself or with ordinary matter. Indeed, that's why it's dark: to emit light it would have to interact via the electromagnetic force.
- Because electromagnetism plays no role in the distribution of dark matter, however, dark matter forms large, smooth, spherical clumps, usually filled by ordinary galaxies plus hot gas or plasma, which it has trapped and retained solely through gravitation.

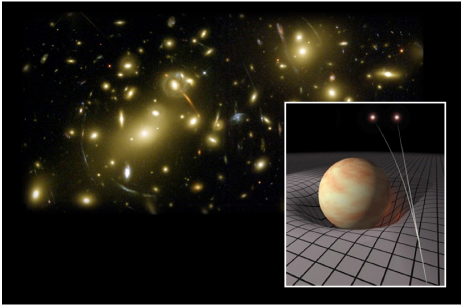
Gravitational Lensing

- Mass curves the space around it, bending the paths along which rays of light travel.



Gravitational Lensing

→ *The more mass and the closer to the center of mass, the more space bends, and the more the image of a distant object is displaced and distorted.*



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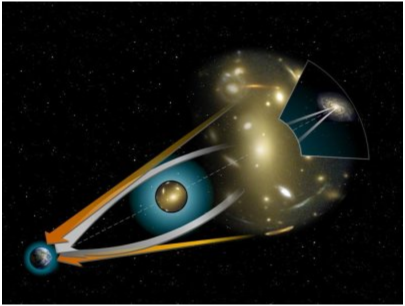
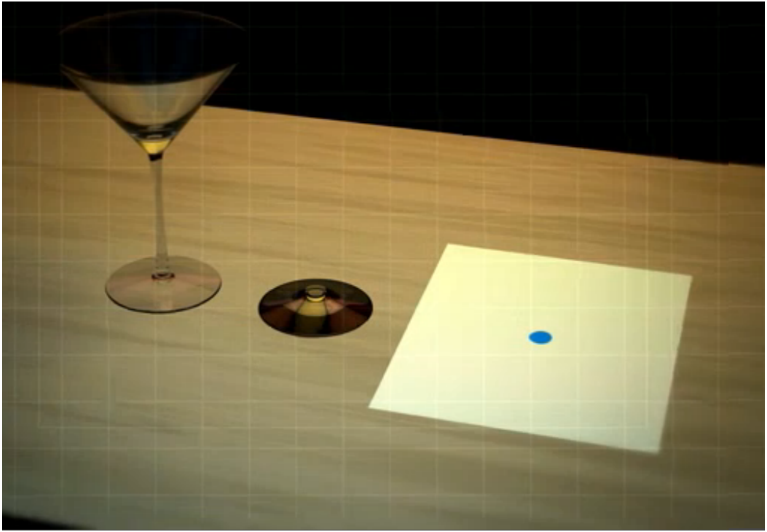


Thus: measuring distortion (or *shear*) is key to measuring the mass of the lensing object itself.

Homemade gravitational lens effect



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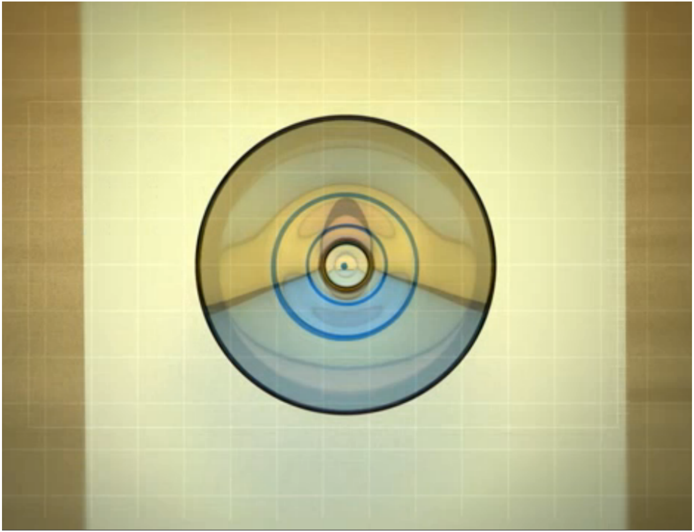

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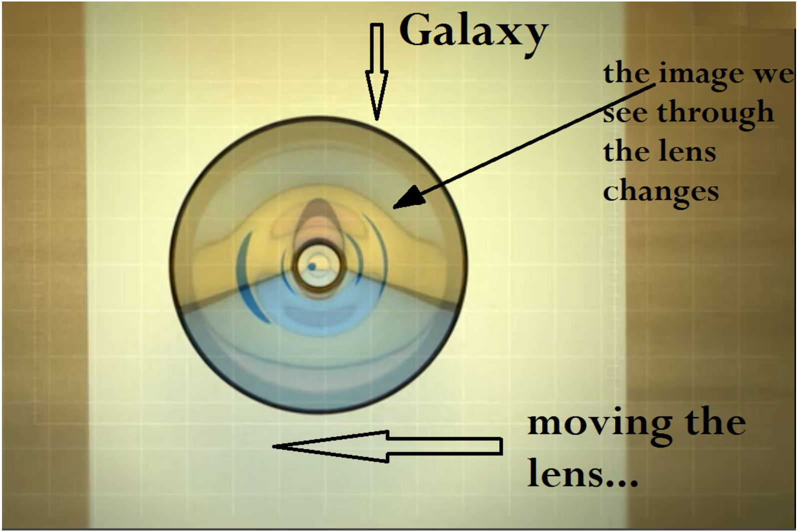
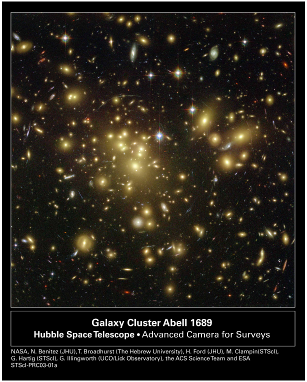


Gravitational Lensing

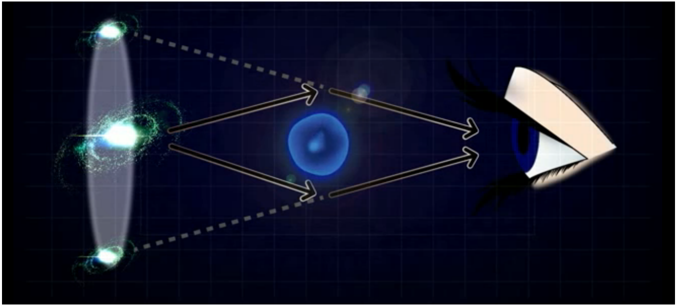


[1]

[1] An international team of astronomers using the NASA/ESA Hubble Space Telescope has discovered a ghostly ring of dark matter that was formed long ago during a titanic collision between two massive galaxy clusters. It is the first time that a dark matter distribution has been found that differs substantially from the distribution of ordinary matter.

Gravitational Lensing - Strong Lensing

Lensing of this type are called *strong* lensing.
(A very massive object or collection of objects, like a nearby galaxy cluster and the dark matter that encloses it).



$\hookrightarrow$ The visible distortion is a direct measure of the mass of the lens and points to its center.

Gravitational Lensing - Weak Lensing

- There are also *weak lensing*, work in the same way of strong lensing, exept that the shear is too subtle to be seen sirectly. Most of the apparent shear isn't distortion at all: a galaxy has its own distinct shape, and we often see it from an angle that makes it too elongated.

  Faint additional distortions in a collection of distant galaxies can be calculated *statistically*, and the average shear due to the lensing of some massive object in front of them can be computed.

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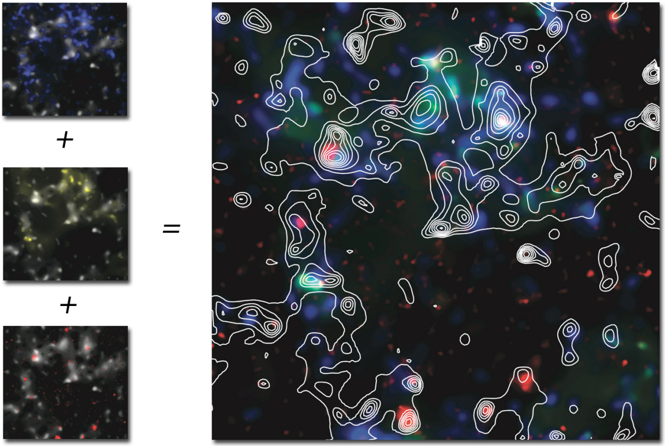
  Faint additional distortions in a collection of distant galaxies can be calculated statistically, and the average shear due to the lensing of some massive object in front of them can be computed.

  Yet, to calculate the mass of the lens from average shear, one has to know its center.

Mapping the Dark Matter



Possible DM Candidates - Barionic Dark Matter

- **MACHOs** (MAssive Compact Halo Objects) :
  - Brown Dwarfs
  - Low mass, Faint red Stars
  - White Dwarfs
  - Neutron Stars
  - Black Holes

$\rightarrow$ Detection Method: *Gravitational Lensing*

Possible DM Candidates - Nonbarionic Dark Matter

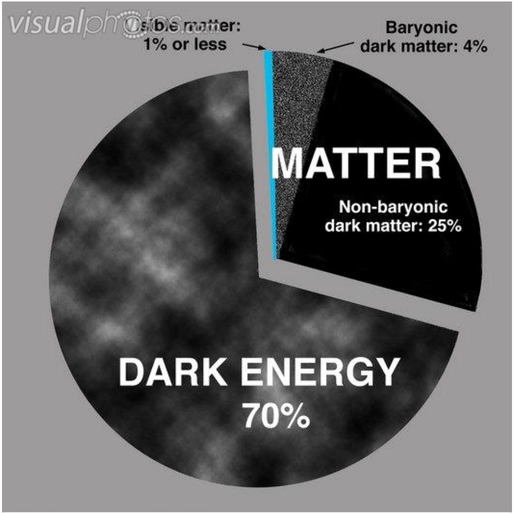
- **WIMPs** (Weakly Interacting Massive Particles) :

  neutral particles formed during the Big Bang, passing through massive particles without interacting and which exert and experience only gravitational (and possibly weak) forces

i.e., photinos, neutrinos, gravitinos, axions... only neutrinos have been detected.

→ Detection Methods and Projects: *The Amanda Project* (Antartica Muon and Neutrino Detector Array), *The Cryogenic Dark Matter Search*, *The DAMA Experiment* (Particle Dark Mattre Searches with Highly Radiopure Scintillators at Gran Sasso).

Composition of the Universe



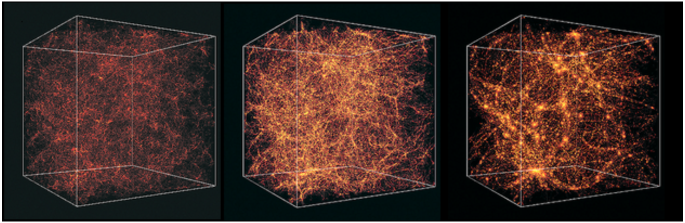
Dark Matter role in Galaxies Formation

- We essentially have accurate data on the distribution of galaxies over most of the evolution of the universe. Although the measurements at earlier epochs have larger errors, due to smaller data sets, their accuracy and power to constrain theoretical models is quite remarkable.

- In 2006, the Chicago scientists based their supercomputer simulations on the assumption that **galaxies form in the center of dark-matter halos.**

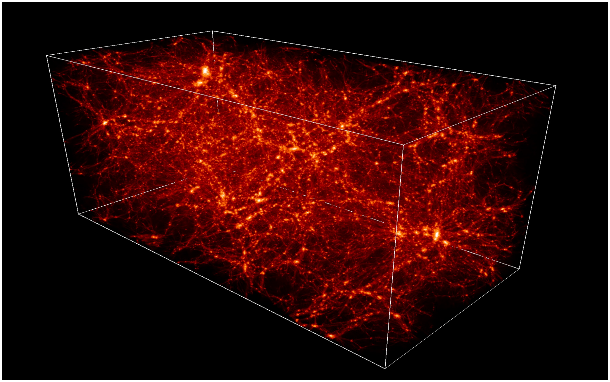
Dark Matter role in Galaxies Formation

- According to this scheme, gravity causes the dark matter in these regions to collapse into halos. These halos provide a central location where normal matter consisting of hydrogen, helium and a small amount of heavier elements would collect in gaseous form. Once this gas had cooled and condensed, it achieved sufficient density for star formation to begin on a galactic scale.

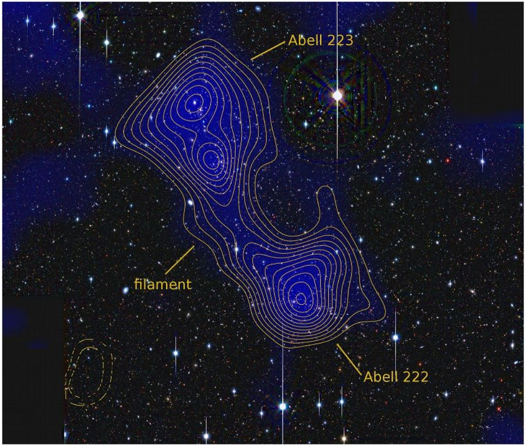


CDM Model - Cluster Formation

CDM Model: asserts that cold dark matter is present in the Universe in the form of long filaments, which build a "*cosmic web*" on a big scale. The model also postulates that in the intersections of these filaments there are cluster of galaxies.



Galaxy Clusters - Filaments of DM



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[2] A July 2012 study of the galaxy clusters Abell 222 and Abell 223 found they are connected by a dark matter filament, shown here. The blue shading and the yellow contours indicate the density of matter.

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