Cepheid Stars as standard candles for distance measurements

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Cepheids are an essential tool for distance measurements in our galactic neighbourhood. This report provides a general overview of the physics of Cepheid stars and its consequences for measuring distances to nearby galaxies. Finally, it includes a method of measuring distances using Cepheids, for high-school students.

Calculating the distance to the galaxy M100 using Cepheids of LMC and M100

In 1597 the first variable star called Mira was discovered by Fabricius. The identification of it and similar stars as variable resulted from brightness measurements. Approximately 200 years later, Pigott and Goodricke found two other stars with a brightness variation. In 1784, Goodricke could detect a period of $5_d\,8_h\,47_{min}$ for δ Cephei's change in brightness. Since δ Cephei was the first variable star which was analyzed in detail, stars of a similar brightness variation are called Cepheids. Later on, Belopolsky could explain the variation in brightness by radius variation of such stars by dint of Doppler effect. He detected a periodic shift of spectral lines of δ Cephei.

Cepheid stars are giants, Bright giants or Supergiants of spectral class F, G or K. Inside the star, Helium is fused to Carbon, Nitrogen and Oxygen. The outside shells consist of HeII-, HeI- and HI. Cepheids pulsate as they are not in a state of hydrostatic equilibrium. The pulsation process can be explained by the so-called Kappa-Mechanism. The fusion of Helium inside the star heats the outside shells. Therefore, the HeII ions are ionized to HeIII, and many electrons are released. That leads to an increase of opacity followed up by an expansion of the star's shells due to the big amount of light pressure from inside. According to the growth of volume, the outside shells cool down and the free electrons are able to recombine with HeIII. Finally, the radiation can escape, the star contracts and the whole process will start again.

We have to take into consideration that the Kappa-mechanism only works for stars being located in the instability strip of the Herzsprung-Russell-Diagram, i.e. in a very small temperature interval. If on the one hand the surface temperature exceeds 7500 K, the He-ionization zone will be too far away from the centre of the star. Thus, the inner mass (the mass between the centre of the star and the He-ionization zone) is too high and the oscillation cannot be efficient. On the other hand, the surface temperature could be lower than 4500 K. Then, the He-ionization zone is too close to the centre of the star and a convective flow of heat of the outside shells prevents the accumulation of energy. Later on, we will rely on that aspect.

In 1912 Henrietta Swan Leavitt who was in search of Cepheid stars in the Magellanic Clouds, discovered a period luminosity relation for Cepheids. She found that Cepheids of a high brightness have higher pulsation periods than those of lower brightness. Current data of the Large Magellantic Cloud (LMC) confirm Leavitt's findings. (Figure 1)

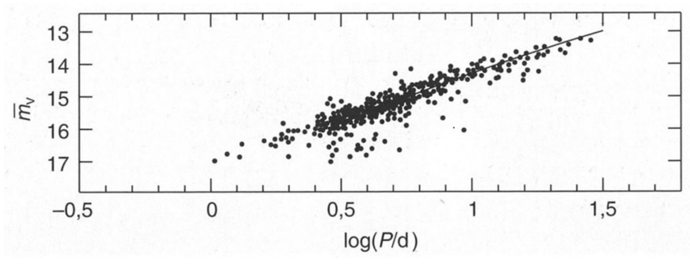

Figure 1: Period-brightness relation for Cepheids in the Large Magellanic Cloud [1]

By dint of Figure 1, we can describe the period-brightness relation for Cepheids in LMC as

$$\bar{m} = -2.765 \log_{10} \frac{P}{\mathrm{d}} + 17.044. \tag{1}$$

To measure distances using the period-luminosity relation of Cepheid stars, some assumptions need to be made.

- The period-luminosity relation of all Cepheids is the same.
- Since Cepheids are only located in the instable zone of the HR-diagram, they all have the same surface temperature.
- All Cepheids have the same internal tructure.
- All Cepheids of one galaxy are equidistant from the Earth.
- The light coming from the Cepheids is not absorbed or reflected by dust clouds. (This assumption is merely necessary for our example.)

In fact, those assumptions are necessary for the following method of distance measurement. Furthermore, they explain where the name *standard candle* originated from. The last assumption is merely important for the example discussed at the end of this report. In modern science the whole procedure appears much more difficult. Nevertheless, the above mentioned assumptions reveal a required comparison of different Cepheids. If you know the distance to a certain Cepheid and ask for the distance to another one, you have to measure the apparent brightnesses of both stars. Now, you can conclude the unknown distance. Generally, astronomers are interested in the distance to a whole galaxy and not just single stars.

Figure 2 shows different period-brightness relations for different galaxies. If all galaxies had the same distance to the earth, all period-brightness relations would be the same. We can make use of the distance modulus for a galaxy i

$$M_{0,i} = m_{0,i} - 5 \log_{10} \frac{r_i}{\mathrm{pc}} + 5.$$

Since the period-luminosity relation of all Cepheids is the same, we know that $M_{0,i}$ and $M_{0,j}$ for two galaxies called i and j is the same,

$$M_{0,i} = M_{0,j}$$

$$m_{0,i} - 5 \log_{10} \frac{r_i}{\mathrm{pc}} + 5 = m_{0,j} - 5 \log_{10} \frac{r_j}{\mathrm{pc}} + 5.$$

If the distance to one galaxy is known, for instance to LMC, the distance to all other galaxies can be calculated from

$$r_j = r_{\mathrm{LMC}} \cdot 10^{\frac{1}{5}(m_{0,j} - m_{0,\mathrm{LMC}})}. \tag{2}$$

If we read off $m_{0,j}$ and $m_{0,\mathrm{LMC}}$ from the period-brightness relations, the distance to galaxy j is easy to find.

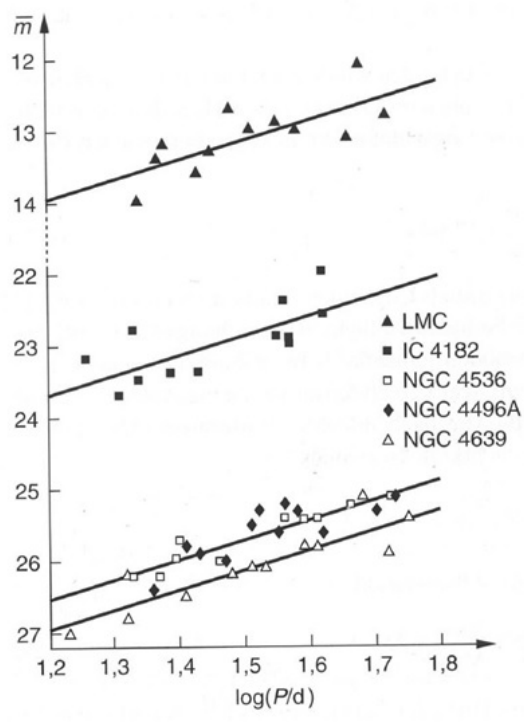

Figure 2: Period-brightness relations of different galaxys [2]

However, this method will only work, if the distance to the LMC is known. Indeed, we could use any other galaxy as a reference, but in 1987 the distance to LMC was measured by dint of the Supernova 1987A.

To work out the method of distance measurements by Cepheid stars the pupils must know all the above mentioned facts. A possible problem might be

"‘Determine the distance to the galaxy M100 using light curves of some Cepheids in that galaxy and in LMC! The distance to the Large Magellanic Cloud is 53.4 kpc.’"

The required light curves are enclosed at the end of this report. The solution of the problem includes the following steps.

1. Measure the maximum and minimum apparent magnitude for each Cepheid!
2. Calculate the average apparent magnitude!
3. Note the pulsation period of each Cepheid!
4. Plot a diagram with the average apparent magnitude on the vertical axis and the decadic logarithm of the period in days on the abscissa!
5. Draw a line through all measure points and find m_0!
6. Do this twice for both M100 and LMC!
7. Use formula (2) and determine the distance to the galaxy M100!

Before outlining the results, the definition of the *average apparent magnitude* deserves consideration. Indeed, the average apparent magnitude is not equal to $\frac{1}{2}(m_{\max} + m_{\min})$. Though, the average luminosity is $\frac{1}{2}(L_{\max} + L_{\min})$. The luminosity and the apparent magnitude of Cepheids are combined by

$$m_2 - m_1 = 2,5 \cdot \log \frac{L_1}{L_2}.$$

Therefore, we can conclude

$$\bar{m} = 2,5 \cdot \log 2 - 2,5 \cdot \log \left(10^{-\frac{2}{5} m_{\max}} + 10^{-\frac{2}{5} m_{\min}} \right). \tag{3}$$

If $\bar{m} = \frac{1}{2}(m_{\max} + m_{\min})$ is used instead of equation (3), the results will only differ a little bit.

Returning to our example, the following results are offered:

$$\begin{aligned} m_{0,\mathrm{M100}} &= 29.68 \\ m_{0,\mathrm{LMC}} &= 18.24 \\ r_{\mathrm{M100}} &= 10.4\,\mathrm{Mpc}. \end{aligned}$$

The distance to the galaxy M100 differs in literature between 11.8 Mpc and 27.7 Mpc. Our result of 10.4 Mpc is near the lower limit of this interval. Taking into account, that the mistakes in measuring the minimum and maximum apparent magnitude are extremely high, our result can be regarded as acceptable.

Cepheid Stars are only useful as standard candles as long as they are visible with a telescope. Therefore, distance measurements by Cepheids are limited to approximately 35 Mpc. Since 35 Mpc is lower than $5 \cdot 10^8$ ly, Cepheids allow distance measurements in our cosmological neighbourhood. Hence, new standard candles, that are much brighter than Cepheids are the Supernovae Ia, which allow distance measurements on a larger scale.

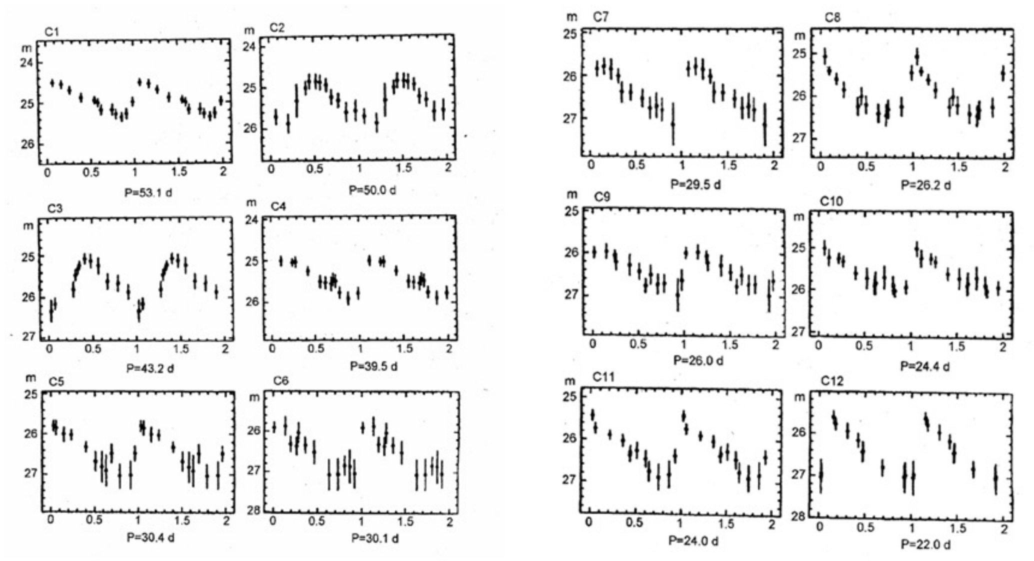

Figure 3: Light Curves of some Cepheids in the galaxy M100 [3]

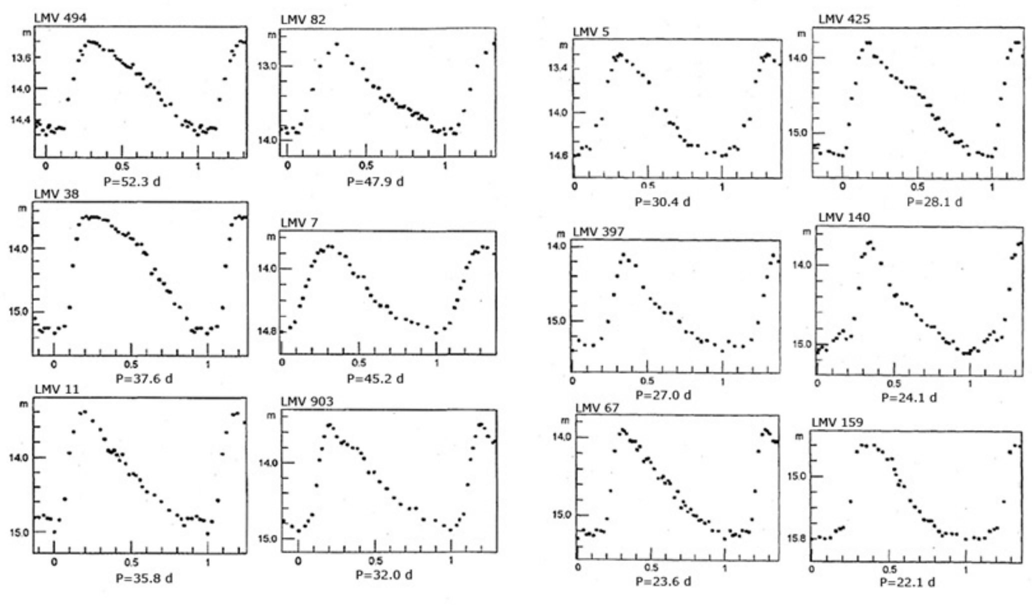

Figure 4: Light Curves of some Cepheids in LMC [3]

References

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List of figures

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