cosmology and gravitational lensing

cosmology lecture (chapter 13)

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Friedmann-Lemaître cosmologies with matter and dark energy for accelerated expansion

thermal history of the universe explains element synthesis and the microwave background

inflation needed for solving the flatness and horizon-problems, and provides Gaussian initial fluctuations for structure growth

formation of the cosmic large-scale structure from inflationary perturbations by gravitational instability

link between statistics and dynamics: linear structure formation is homogeneous (growth equation $D_+(a)$) and conserves Gaussianity of the initial conditions

halo formation: Jeans-criterion for baryons

halo density and merging activity determined by Press-Schechter formalism
gravitational lensing: overview

- gravitational light deflection: test of general relativity (1919)
- strong lensing: giant luminous arcs in clusters of galaxies
- weak lensing: correlated distortion of background galaxy images
- multiply imaged quasars and time delays
- lensed light curves of bulge stars and search of MACHOs
- lensing of the microwave background (2007)
- lensing of the microwave background polarisation (2013/2014)
lensing on a point mass

- gravitational fields $\Phi$ influence the propagation of light: **Shapiro delay**
  \[
  \Delta t = \int \frac{2}{c^3} \Phi \quad (1)
  \]
  light travels slower in a gravitational potential
- we can assign an **index of refraction** to a potential
  \[
  n = 1 - \frac{2}{c^2} \Phi \quad (2)
  \]
  so that the effective speed is $c/n = c - 2\Phi/c$
- we expect lensing effects on gravitational fields due to **Fermat’s principle**
  \[
  \hat{\alpha} = -\int dx \nabla \perp n = \frac{2}{c^2} \int dx \nabla \perp \Phi \quad (3)
  \]
lensing on a point mass

- example: gravitational field of a point mass $M$ at distance $b, z$

$$\Phi(b, z) = -\frac{GM}{\sqrt{b^2 + z^2}} \quad (4)$$

- gradient of the potential

$$\nabla_\perp = \frac{GM}{(b^2 + z^2)^{3/2}} b \quad (5)$$

where $b$ points towards the mass and is perpendicular to the ray

- deflection angle:

$$\hat{\alpha} = \frac{2}{c^2} \int dz \nabla_\perp \Phi = \frac{4GM}{c^2 b} \quad (6)$$
weak perturbations of the metric

• consider Minkowski-line element, weakly perturbed by static gravitational potential $\Phi$

\[ (ds)^2 = \left( 1 + \frac{2}{c^2} \Phi \right) c^2 dt^2 - \left( 1 - \frac{2}{c^2} \Phi \right) d\vec{x}^2 \]  \hspace{1cm} (7)

• on a geodesic, the line element vanishes: derive effective index of refraction $n$

\[ \frac{d|\vec{x}|}{dt} = c' = \frac{c}{n} \text{ with } n = 1 - \frac{2}{c^2} \Phi \]  \hspace{1cm} (8)

• Fermat’s principle: photon minimises run time $\int |d\vec{x}| n$

\[ \delta \int_{x_i}^{x_f} ds \sqrt{\frac{d\vec{x}^2}{ds^2} n(\vec{x}(s))} = 0, \]  \hspace{1cm} (9)

for parametrisation $x(s)$ of trajectory with $|d\vec{x}/ds| = 1$
lens equation

- carry out the variation yields ($\nabla_\perp = \nabla - \vec{e}(\vec{e}\nabla)$):
  \[
  \nabla n - \vec{e}(\vec{e}\nabla n) - n \frac{d\vec{e}}{ds} = 0 \rightarrow \frac{d\vec{e}}{ds} = \nabla_\perp \ln n \simeq -\frac{2}{c^2} \nabla_\perp \Phi \quad (10)
  \]

- deflection $\hat{\alpha} = \vec{e}_f - \vec{e}_i = -\frac{2}{c^2} \int ds \nabla_\perp \Phi$

- read off lens equation, use deflection angle $\hat{\alpha}$:
  \[
  \tilde{\eta} = \frac{D_s}{D_l} \xi - D_{ls} \hat{\alpha} \rightarrow \beta = \theta - \frac{D_{ls}}{D_s} \hat{\alpha}(\theta) = \theta - \tilde{\alpha} \quad (11)
  \]
approximations

- formally: \( \hat{\alpha} = \vec{\varepsilon}_f - \vec{\varepsilon}_i = -\frac{2}{c^2} \int ds \nabla_\perp \Phi \)
- nonlinear integral: the deflection determines the path on which one needs to carry out the integration
- **Born-approximation**: integration along a fiducial straight ray instead of actual photon geodesic
- if the travel path (of order \( c/H_0 \)) is large compared to the size of the lens, then the gravitational interaction can be taken to be instantaneous → **thin-lens approximation**
- in this case: project the surface mass density \( \Sigma \)

\[
\Sigma(\vec{b}) = \int dz \, \rho(\vec{b}, z) \quad (12)
\]

- deflection is the superposition of all surface density elements

\[
\hat{\alpha}(\vec{b}) = \frac{4G}{c^2} \int d^2b' \, \Sigma(\vec{b}') \frac{\vec{b} - \vec{b}'}{|\vec{b} - \vec{b}'|^2} \quad (13)
\]
Einstein radius of a gravitational lens

- Einstein ring: look at deflection

\[ \beta = \theta - \alpha = \theta - \frac{D_{ds}}{D_d D_s} \frac{4GM}{c^2 \theta} \]  \hfill (14)

- if the source lies on the optical axis (\( \beta = 0 \)) and if the lens is massive enough all light rays are focused

- we can compute the radius of the ring (in angular units)

\[ \theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{ds}}{D_d D_s}} \]  \hfill (15)

which is called the **Einstein**-radius
strong lensing and Einstein-rings

Einstein ring around an elliptical galaxy, source: SLACS survey

- perfect alignment of source and lens give rise to **Einstein rings**
lens mapping and the mapping Jacobian

- lens equation $\beta = \theta - \vec{\alpha}(\theta)$ relates true position $\theta$ to observed position $\beta$ with mapping field $\alpha$
- if mapping $\alpha = \nabla_\perp \psi$ is not constant across galaxy image $\rightarrow$ distortion of observed shape
- describe with Jacobian-matrix $J$

$$J = \frac{\partial \beta}{\partial \theta} = \left( \delta_{ij} - \frac{\partial^2 \psi(\theta)}{\partial \theta_i \partial \theta_j} \right)$$ (16)

- decompose $A = \text{id} - J$ in terms of Pauli-matrices:

$$A = \sum_{\alpha} a_\alpha \sigma_\alpha = \kappa \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \gamma_+ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \gamma_\times \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$ (17)

- coefficients: $\kappa$ (convergence), $\gamma_+$ and $\gamma_\times$ (shear)
- combine shear coefficients to complex shear $\gamma = \gamma_+ + i\gamma_\times$ (spin 2)
image distortions

- deflection not observable, actual position of a galaxy is unknown
- with assumptions on galaxy ellipticity, the shearing is observable
- bending of an image (flexion) is a new lensing method

question

why is there no rotation of a galaxy image in lensing?
analytical profiles: singular isothermal sphere

- from the part about the stability of self-gravitating systems we know the singular isothermal sphere:

  \[ \rho(r) = \frac{\sigma_v^2 \nu^2}{2\pi G} \times \frac{1}{r^2} \]  
  \hspace{1cm} (18)

where the unordered particle motion is described by the velocity dispersion \( \sigma_v^2 \)

- compute surface mass density by projection

  \[ \Sigma(x) = \frac{\sigma_v^2}{2G} \times \frac{1}{x} \]  
  \hspace{1cm} (19)

- from which we get the deflection angle

  \[ \hat{\alpha} = 4\pi \frac{\sigma_v^2}{c^2} \]  
  \hspace{1cm} (20)
mass reconstructions

- convergence $\propto$ local surface mass density $\Sigma$ of a lens
- but: it is **not directly observable** → is it possible to infer $\kappa$ and the mass map from the observation of gravitational shear?
- write down derivative relations in Fourier space
  \[
  \kappa = -\frac{1}{2}(k_x^2 + k_y^2)\psi
  \quad \gamma_+ = -\frac{1}{2}(k_x^2 - k_y^2)\psi
  \quad \gamma_\times = -k_x k_y \psi
  \]  
  (21)
- combine into single equation
  \[
  \begin{pmatrix}
  \gamma_+ \\
  \gamma_\times 
  \end{pmatrix} = \frac{1}{k^2} \begin{pmatrix}
  k_x^2 - x_y^2 \\
  2k_x k_y
  \end{pmatrix} \kappa
  \]  
  (22)
- operator is **orthogonal**: $A^2 = \text{id}$
  \[
  \left[ \frac{1}{k^2} \begin{pmatrix}
  k_x^2 - k_y^2 \\
  2k_x k_y
  \end{pmatrix} \right]^2 = 1
  \]  
  (23)
example: cluster profiles

\[ \kappa = \frac{1}{k^2} \left[ (k_x^2 - k_y^2)\gamma_+ + 2k_x k_y \gamma_x \right] \]
yields estimate of map \( \Sigma \)

question

derive the reconstruction operator in real space and formulate the inversion as an integration, identify the Green-function
lensing on the large-scale structure: fluctuation statistics of the lensing signal reflects the fluctuation statistics of the density field

neighboring galaxies have correlated deformations because the light rays cross similar, correlated tidal fields
tidal fields and their effect on light rays

• distance $x$ of a gravitationally deflected light ray relative to a fiducial straight line is

$$\frac{d^2 x}{d\chi^2} = -\frac{2}{c^2} \nabla_{\perp} \Phi$$  \hspace{1cm} (24)

• solution (flat universes)

$$x = \chi \theta - \frac{2}{c^2} \int d\chi' (\chi - \chi') \nabla_{\perp} \Phi(\chi' \theta)$$  \hspace{1cm} (25)

• deflection angle

$$\alpha = \frac{\chi \theta - x}{\chi} = \frac{2}{c^2} \int d\chi' \frac{\chi - \chi'}{\chi} \nabla_{\perp} \Phi(\chi' \theta)$$  \hspace{1cm} (26)

• convergence, with $\nabla_{\theta} = \chi \nabla_{\chi}$

$$\kappa = \frac{1}{2} \text{div} \alpha = \frac{1}{c^2} \int d\chi' (\chi - \chi') \frac{\chi'}{\chi} \Delta \Phi(\chi' \theta)$$  \hspace{1cm} (27)
tidal fields and their effect on light rays

- relate to density field with (comoving) Poisson-equation

$$\Delta \Phi = \frac{3H_0^2\Omega_m}{2a} \delta$$  \hspace{1cm} (28)

- final result:

$$\kappa = \int d\chi' W(\chi, \chi') \delta \quad \text{with} \quad W(\chi, \chi') = \frac{3}{2} \left( \frac{H_0}{c} \right)^2 \frac{\Omega_m}{a} (\chi - \chi') \frac{\chi'}{\chi}$$  \hspace{1cm} (29)

- fluctuations in $\kappa$ reflect fluctuations in $\delta$ **in a linear way**

**cosmic shear**

gravitational shear of a galaxy measures the integrated matter density along the line of sight, weighted by $W(\chi)$
ray-tracing simulations of weak lensing

- solve transport \( \frac{d^2}{dw^2} x = -\frac{2}{c^2} \nabla_\perp \Phi \) by discretisation

source: C. Pfrommer
simulated shear field on an $n$-body simulation

- Gadget-simulated, side length $100 \text{ Mpc}/h$, 40 planes
- clusters of galaxies produce characteristic pattern in shear field
Limber-equation

- original title: Limber (1953), *The Analysis of Counts of the Extragalactic Nebulae in Terms of a Fluctuating Density Field*

- **relate** 3d-power spectrum $P(k)$ to observed 2d-power spectrum $C(\ell)$

- define correlation function $C(\theta) = \langle g(\theta_1)g(\theta_2) \rangle$ of quantity $g$, which measures fluctuations in density field $g(\theta) = \int d\chi W(\chi)\delta(\chi\theta,\chi)$

- assume that weighting function $q(\chi)$ does not vary much compared to fluctuation scale:

\[
C(\theta) = \int d\chi W(\chi)^2 \int d(\Delta\chi) \xi\left(\sqrt{(\chi\theta)^2 + \Delta^2\chi,\chi}\right)
\]  

(30)

- correlation function $C(\theta)$ can be Fourier-transformed to yield angular power spectrum $C(\ell)$:

\[
C(\ell) = \int d\chi \frac{W(\chi)^2}{\chi^2} P\left(k = \frac{\ell}{\chi},\chi\right)
\]  

(31)
• the spectrum $P(k)$ is defined as
\[
\langle \delta(k)\delta(k') \rangle = (2\pi)^3 \delta_D(k + k')P(k)
\]
from the Fourier-transform of the density field $\delta(x)$
\[
\delta(k) = \int d^3x \, \delta(x) \exp(-i k x) \quad \leftrightarrow \quad \delta(x) = \int \frac{d^3k}{(2\pi)^3} \, \delta(k) \exp(+i k x)
\]

• if the field is not defined in Cartesian coordinates but exists on the surface of the sphere (like an observation at a position on the sky), one needs to use spherical harmonics for decomposition:
\[
\gamma(\hat{\theta}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{+\ell} \gamma_{\ell m} Y_{\ell m}(\hat{\theta}) \quad \leftrightarrow \quad \gamma_{\ell m} = \int_{4\pi} d\Omega \, \gamma(\hat{\theta}) Y^*_{\ell m}(\hat{\theta})
\]
and the spectrum reads:
\[
\langle \gamma_{\ell m} \gamma^*_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{mm'} C(\ell)
\]
**Limber-equation: correlation functions**

- Observable: shear $\gamma$ at position $\hat{\theta}$ on the sky:

  $$\gamma(\hat{\theta}) = \int_0^{\chi_H} d\chi \, W_\gamma(\chi) \delta(\chi \hat{\theta}, \chi)$$  \hspace{1cm} (36)

- Write down correlation function as the Fourier-transform of $P(k)$ and project:

  $$C_{\gamma\gamma}(\alpha) = \int_0^{\chi_H} d\chi W_\gamma(\chi) \int_0^{\chi_H} d\chi' W_\gamma(\chi') \int dk k^2 P(k, \chi, \chi') \int_{4\pi} d\Omega_k \exp(ik(x - x'))$$  \hspace{1cm} (37)

- Correlation function as the Fourier-transform of the spectrum

  $$\langle \gamma(\hat{\theta} \chi, \chi) \gamma^*(\hat{\theta}' \chi', \chi') \rangle = \int \frac{d^3 k}{(2\pi)^3} P(k) \exp(ik(x - x'))$$  \hspace{1cm} (38)

- With the integration done in spherical coordinates

  $$\langle \gamma(\hat{\theta} \chi, \chi) \gamma^*(\hat{\theta}' \chi', \chi') \rangle = \int dk k^2 P(k) \int_{4\pi} d\Omega_k \exp(ik(x - x'))$$  \hspace{1cm} (39)
### Limber-equation: Rayleigh-decomposition

- Rayleigh: decomposition of plane waves in spherical waves

\[
\exp(ikx) = 4\pi \sum_{\ell=0}^{\infty} i^\ell \tilde{j}_\ell(kx) \sum_{m=-\ell}^{+\ell} Y_{\ell m}(\hat{k})Y_{\ell m}^*(\hat{\theta}) \tag{40}
\]

- rewrite Fourier-waves as spherical waves:

\[
\int d\Omega_k \exp(ik(x-x')) = (4\pi)^2 \sum_{\ell=0}^{\infty} j_\ell(k\chi)j_\ell(k\chi') \sum_{m=-\ell}^{+\ell} Y_{\ell m}(\hat{\theta})Y_{\ell m}^*(\hat{\theta}') \tag{41}
\]

- use addition theorem of spherical harmonics

\[
\int d\Omega_k \exp(ik(x-x')) = 4\pi \sum_{\ell=0}^{\infty} j_\ell(k\chi)j_\ell(k\chi') (2\ell + 1) P_\ell(\cos \alpha) \tag{42}
\]

- write correlation function \( C_{\gamma\gamma}(\alpha) \) from \( P(k) \)

\[
C_{\gamma\gamma}(\alpha) = 4\pi \int_0^{\chi_H} d\chi W_\gamma(\chi) \int_0^{\chi_H} d\chi' W_\gamma(\chi') \int dk k^2 P(k,\chi,\chi') \sum_{\ell=0}^{\infty} j_\ell(k\chi)j_\ell(k\chi') \tag{43}
\]
Limber-equation: angular spectra

- transform correlation function to $\ell$-space by Fourier-transform
  \[ C_{\gamma\gamma}(\ell) = (4\pi)^2 \int_0^{X_H} d\chi W_\gamma(\chi) \int_0^{X_H} d\chi' W_\gamma(\chi') \int dk k^2 P(k, \chi, \chi') j_\ell(k\chi) j_\ell(k\chi') \]  
  (44)

- use orthonormality of spherical Bessel functions
  \[ \int_0^\infty k^2 dk j_\ell(k\chi) j_\ell(k\chi') = \frac{\pi}{2\chi^2} \delta_D(\chi - \chi') \]  
  (45)

- Bessel-functions sort out $P(k) \approx P(\ell/\chi)$, such that:
  \[ C_{\gamma\gamma}(\ell) \approx \int_0^{X_H} \frac{d\chi}{\chi^2} W^2_\gamma(\chi) P(k = \ell/\chi, \chi) \]  
  (46)

**Limber-equation**
relates fluctuation statistics of the 3d-source field to the statistics of the 2d projected observable
Limber-equation: additional formulas

• angular spectrum from the correlation function

\[ C_{\gamma\gamma}(\ell) = 2\pi \int d\cos\alpha \, C_{\gamma\gamma}(\alpha)P_\ell(\cos\alpha) \] (47)

• correlation function from the angular spectrum

\[ C_{\gamma\gamma}(\alpha) = \frac{1}{4\pi} \sum_{\ell=0}^{\infty} (2\ell + 1)C_{\gamma\gamma}(\ell)P_\ell(\cos\alpha) \] (48)

• addition theorem of the spherical harmonics

\[ \sum_{m=-\ell}^{+\ell} Y_{\ell m}(\hat{\theta})Y_{\ell m}^*(\hat{\theta}') = \frac{2\ell + 1}{4\pi} P_\ell(\cos\alpha) \] (49)
shear power spectra

- use Limber’s equation to link the shear power spectrum to the dark matter power spectrum
- cosmology: redshift weightings $W(\chi)$, growth $D_+(a(\chi))$, normalisation reflects $\sigma_8$

source: Bartelmann & Schneider, physics reports 340 (2001)
shear in apertures

source: Bartelmann & Schneider, physics reports 340 (2001)

- improve constraint on $\sigma_8$: $C(\ell)$ should be determined by a small range of $k$-modes
- average $\gamma$ in an aperture of size $\theta$: $\langle |\gamma|^2 \rangle (\theta)$: product in $\ell$-space

$$\langle |\gamma|^2 \rangle (\theta) = 2\pi \int_0^\infty \ell \, d\ell \, C_\gamma(\ell) \left[ \frac{J_1(\theta \ell)}{\pi \theta \ell} \right]^2$$

(50)
parameter estimates from weak cosmic shear

\[ \Omega_{EDE} \] and \( w_0 \), source: L. Hollenstein

- lensing is a powerful method for determining parameters
- even complicated dark energy models can be investigated
future lensing surveys

- coverage $\sim$ half of the sky, going to unit redshift
- precision determination of cosmological parameters, statistical errors $\sim 10^{-3\ldots-4}$
- challenge: **systematics control**
weak lensing tomography
measurements of galaxy shapes

- observe distortion in the shape of lensed galaxies
- measure second moments of brightness distribution

\[ Q_{ij} = \frac{\int d^2\theta I(\theta)(\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int d^2\theta I(\theta)} \] (51)

- define complex ellipticity (spin 2):

\[ \epsilon = \frac{Q_{xx} - Q_{yy} + 2iQ_{xy}}{Q_{xx} + Q_{yy} + 2 \sqrt{Q_{xx}Q_{yy} - Q_{xy}^2}} \] (52)

- mapping of complex ellipticity by a Jacobian with reduced shear

\[ g(\theta) = \frac{\gamma(\theta)}{1 - \kappa(\theta)} \]

\[ \epsilon = \frac{\epsilon' + g}{1 + g^*\epsilon'} \text{ for } |g| \leq 1, \quad \epsilon = \frac{1 + (\epsilon')^*g}{(\epsilon')^* - g'} \text{ for } |g| > 1 \] (53)
galaxy shapes with shapelets

shapelet base functions $B_{ij}$, source: P. Melchior

- decomposition into a set of basis functions based on the quantum mechanical harmonic oscillator: Hermite polynomials
lensing of the cosmic microwave background

- weird (non-Gaussian) patterns in the deflection field
- measurement of lensing at high redshift, in temperature and polarisation
lensed and unlensed CMB spectra, source: Ph. Merkel

- lensing wipes out structures in the CMB (compare to frosted glass)
- amplitudes of the CMB spectrum decreases, non-Gaussianitites in the CMB are generated
- polarisation correlations more strongly affected, $B$-modes
microlensing and MACHOs

- compact massive objects (historical dark matter candidates) orbit the Milky Way
- observe a large number of bulge stars or stars in the LMC
- find lensed light curves, very typical signature

source: C. Alcock
time delay measurements with quasars

- image appears if the variation of the gravitational time delay is zero
- time delays between different images differ by days
- geometry of the lens can be determined, including the distance

source: universe review
the deflection can be written as the gradient of the lensing potential

$$\theta - \beta = \nabla \psi \quad (54)$$

which can be combined into a single condition

$$\nabla \left( \frac{1}{2} (\theta - \beta)^2 - \psi \right) = 0 \quad (55)$$

compare with time-delay function

$$\Delta t(\theta) = \frac{1 + z}{c} \frac{D_d D_s}{D_{ds}} \left( \frac{1}{2} (\theta - \beta)^2 - \psi \right) = \Delta t_{\text{geo}} + \Delta t_{\text{grav}} \quad (56)$$

the first term corresponds to the time delay along the lensed trajectory, the second term is the Shapiro delay in a gravitational potential

Fermat’s principle now requires $$\nabla \Delta t(\theta) = 0$$, which might have multiple solutions
Summary: Friedmann-Lemaître Cosmologies

- **Dynamic** world models based on general relativity
- Robertson-Walker line element as a solution to the field equation
- Copernican principle: homogeneous and isotropic metric
- Homogeneous fluids, with a certain pressure density relation, parameterised by $w = p/\rho$
  - Radiation ($w = 1/3$)
  - (Dark) matter ($w = 0$)
  - Curvature ($w = -1/3$)
  - Cosmological constant ($w = -1$)
- Hubble parameter $H_0$ defines the critical density $\rho_{\text{crit}} = 3H_0^2/(8\pi G)$
- Distance definitions become ambiguous
- Geometrical probes constrain the model parameters to a few percent, in particular $\Omega_k < 0.01$
summary: random fields and spectra

- inflation: epoch of rapid **accelerated** expansion of the early universe
  - Hubble expansion dominated by a fluid with very negative $w$
    - drives curvature towards zero $\rightarrow$ flatness problem
    - grows observable universe from a small volume $\rightarrow$ horizon problem
- fluctuations in the energy density of the inflaton field couple gravitationally to the other fluids
- fluctuations are Gaussian and have a finite correlation length
  - characterisation with a correlation function $\xi(r)$
  - homogeneous fluctuations: spectrum $P(k)$
- inflationary fluctuations can be observed as temperature anisotropies in the CMB
- shape of the spectrum: inflation gives $P(k) \propto k^{n_s}$, changed by transfer function $T(k)$ in the Meszaros effect, normalised by $\sigma_8$
summary: structure formation

• cosmic structures and the large-scale distribution of galaxies form by gravitational instability of inflationary perturbation
  • continuity equation
  • Euler equation
  • Poisson equation
• linearisation for small amplitudes: homogeneous growth, described by $D_+(a)$, conservation of Gaussianity of initial conditions
• nonlinear growth is inhomogeneous and destroys Gaussianity by mode coupling
• three basic difficulties
  • nonlinearities in the continuity and Euler-equation
  • collisionlessness of dark matter
  • non-extensivity of gravity
• galaxy formation: gravitational collapse, Jeans argument
• halo density: predicted from $P(k)$ with Press-Schechter formalism
summary: standard model ΛCDM

• ΛCDM is a flat, accelerating Friedmann-Lemaître cosmology with dark matter and a cosmological constant

• ΛCDM has 7 parameters, and is in remarkable agreement with observations, both of geometrical and growth probes

1. \( \Omega_m = 0.25 \), low density, required by supernova observations
2. \( \Omega_b = 0.04 \), small value, good measurement from CMB
3. \( \Omega_\Lambda = 0.75 \), flatness from CMB, \( \Omega_m + \Omega_\Lambda = 1 \)
4. \( w = -1 \), cosmological constant, no dynamic dark energy
5. \( \sigma_8 = 0.8 \), low value (compared to history), largest uncertainty
6. \( n_s = 0.96 \), predicted by inflation to be \( \lesssim 1 \)
7. \( h = 0.72 \), sets expansion time scale, or age/size of the universe

• up to now, there is no theoretical understanding of Λ or of the magnitude of \( H_0 \)
summary: open questions in cosmology

- precision determination of cosmological parameters and verification of the standard model
- matter content of the Universe: dark matter particles, cosmological neutrinos
- inflation, conditions for inflation and observables, Gaussianity
- gravitational waves in the early universe
- quantification of the nonlinearly evolved cosmic density field, description of nonlinear structure formation processes
- substructure of dark matter haloes and an explanation of their kinematic structure
- biasing of galaxies and relations between host halo properties and member galaxies, galaxy formation and evolution
- distinguishing between cosmological constant $\Lambda$, dark energy or modified gravity
- tidal interactions of haloes with the large-scale structure