formation of the cosmic large-scale structure

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outline

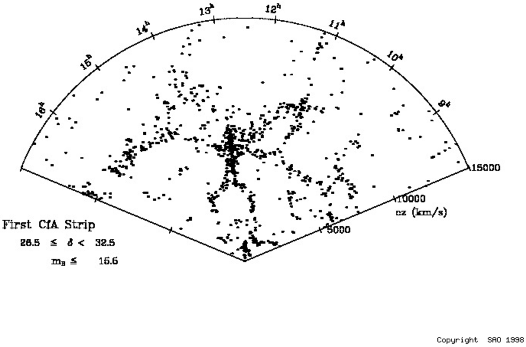
structures

up to now

we always worked with homogeneously distributed matter (in the context of FLRW-cosmologies). but look at all the structure!

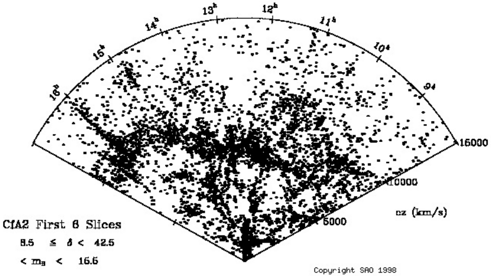
- mean density of the Universe: $\rho \simeq 10^{-29}\mathrm{g/cm^3}$
- density inside a cluster of galaxies: $\rho \simeq 10^{-27}\mathrm{g/cm^3}$
- density inside a galaxy: $\rho \simeq 10^{-29}\mathrm{g/cm^3}$
- density of the Earth: $\rho \simeq 5\mathrm{g/cm^3}$
- density of the Sun: $\rho \simeq 1.5\mathrm{g/cm^3}$
- density of a white dwarf: $\rho \simeq 10^{6}\mathrm{g/cm^3}$
- density of a Neutron star: $\rho \simeq 10^{14}\mathrm{g/cm^3}$

filamentary structures: the stickman



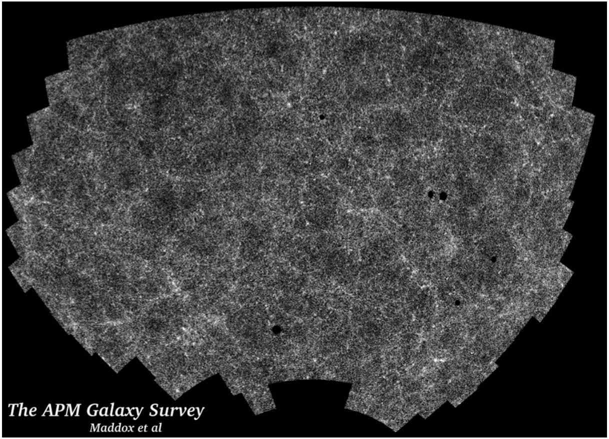
distribution of galaxies (source: CFA, Harvard)

filamentary structures: the great wall



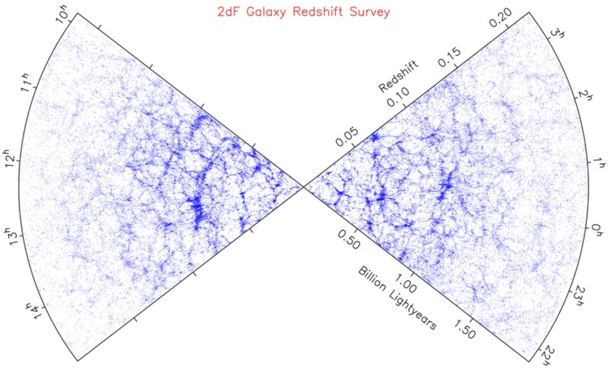
distribution of galaxies (source: CFA, Harvard)

large-scale structure: APM survey



distribution of galaxies (source: APM survey)

large-scale structure: 2dF survey



distribution of galaxies (source: 2dF survey)

properties of dark matter

current paradigm:

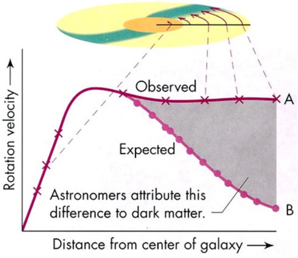
structures from by gravitational instability from inflationary fluctuations in the cold dark matter (CDM) distribution

- collisionless → very small interaction cross-section
- cold → negligible thermal motion at decoupling, no cut-off in the spectrum $P(k)$ on a scale corresponding to the diffusion scale
- dark → no interaction with photons, possible weak interaction
- matter → gravitationally interacting

main conceptual difficulties

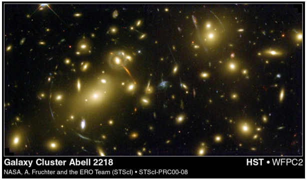
- collisionlessness → hydrodynamics, no pressure or viscosity
- non-saturating interaction (gravity) → extensivity of binding energy

galaxy rotation curves



- balance centrifugal and gravitational force
- stars move too fast if only visible matter generates the potential
- there must be invisible matter in the galaxy

virial equilibrium of clusters



source: NASA

- compare velocities and depth of gravitational potential
- galaxies move too fast if only visible matter would produce the potential
- there must be invisible matter around (or the virial theorem would be wrong, or the gravitational potential would be different)

dark matter and the microwave background

- fluctuations in the density field at the time of decoupling are $\simeq 10^{-5}$
- long-wavelength fluctuations grow proportionally to a
- if the CMB was generated at $a = 10^{-3}$, the fluctuations can only be 10^{-2} today
- large, supercluster-scale objects have $\delta \simeq 1$

cold dark matter

need for a **nonbaryonic** matter component, which is not interacting with photons

structure formation equations

cosmic structure formation

cosmic structures are generated from tiny inflationary seed fluctuations, as a fluid mechanical, self-gravitating phenomenon (with Newtonian gravity), on an expanding background

- continuity equation: no matter ist lost or generated

$$\frac{\partial}{\partial t}\rho + \mathrm{div}(\rho\vec{\upsilon}) = 0 \tag{1}$$

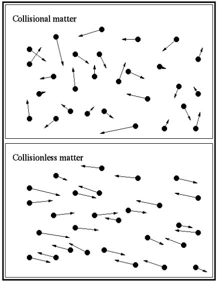
- Euler-equation: evolution of velocity field due to gravitational forces

$$\frac{\partial}{\partial t}\vec{\upsilon} + \vec{\upsilon}\nabla\vec{\upsilon} = -\nabla\Phi \tag{2}$$

- Poisson-equation: potential is sourced by the density field

$$\Delta\Phi = 4\pi G\rho \tag{3}$$

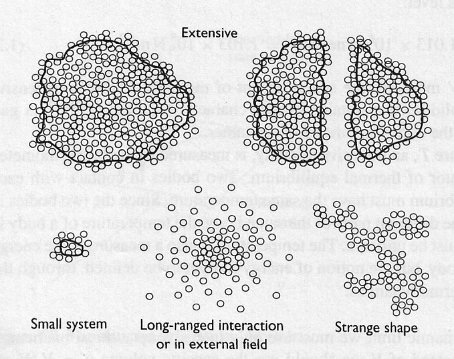
collisionlessness of dark matter



source: P.M. Ricker

- CDM is collisionless (elastic collision cross section $\ll$ neutrinos)
  - why can galaxies rotate and how is vorticity generated?
  - why do galaxies form from their initial conditions without viscosity?
  - how can one stabilise galaxies against gravity without pressure?
  - is it possible to define a temperature of a dark matter system?

non-extensivity of gravity



source: Kerson Huang, statistical physics

- gravitational interaction is long-reached
- gravitational binding energy per particle is not constant for $n \to \infty$

structure formation equations

cosmic structure formation

structure formation is a self gravitating, fluid mechanical phenomenon

- continuity equation: evolution of the density field due to fluxes

$$\frac{\partial}{\partial t}\rho + \mathrm{div}(\rho\vec{\upsilon}) = 0 \tag{4}$$

- Euler equation: evolution of the velocity field due to forces

$$\frac{\partial}{\partial t}\vec{\upsilon} + \vec{\upsilon}\nabla\vec{\upsilon} = -\nabla\Phi \tag{5}$$

- Poisson equation: potential sourced by density field

$$\Delta\Phi = 4\pi G\rho \tag{6}$$

- 3 quantities, 3 equations → solvable
- 2 nonlinearities: $\rho\vec{\upsilon}$ in continuity and $\vec{\upsilon}\nabla\vec{\upsilon}$ in Euler-equation

regimes of structure formation

look at overdensity field $\delta \equiv (\rho - \bar{\rho})/\bar{\rho}$, with $\bar{\rho} = \Omega_m \rho_{\mathrm{crit}}$

- analytical calculations are possible in the regime of linear structure formation, $\delta \ll 1$
  $\rightarrow$ homogeneous growth, dependence on dark energy, number density of objects
- transition to non-linear structure growth can be treated in perturbation theory (difficult!), $\delta \sim 1$
  $\rightarrow$ first numerical approaches (Zel'dovich approximation), directly solvable for geometrically simple cases (spherical collapse)
- non-linear structure formation at late times, $\delta > 1$
  $\rightarrow$ higher order perturbation theory (even more difficult), ultimately: direct simulation with n-body codes

linearisation: perturbation theory for $\delta \ll 1$

- move from physical to comoving frame, related by scale-factor a
- gradients: $\nabla \rightarrow a^{-1}\nabla$, Laplace: $\Delta \rightarrow a^{-2}\Delta$
- use density $\delta = \Delta\rho/\rho$ and comoving velocity $\vec{u} = \vec{\upsilon}/a$
  - **linearised continuity equation**:

$$\frac{\partial}{\partial t}\delta + \mathrm{div}\vec{u} = 0$$

  - **linearised Euler equation**: evolve momentum

$$\frac{\partial}{\partial t}\vec{u} + 2H(a)\vec{u} = -\frac{\nabla\Phi}{a^2}$$

  - **Poisson equation**: generate potential

$$\Delta\Phi = 4\pi G\rho_0 a^2\delta$$

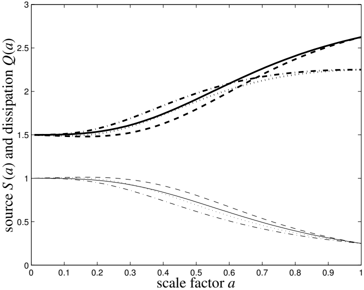
growth equation

- structure formation is homogeneous in the linear regime, all spatial derivatives drop out
- combine continuity, Jeans- and Poisson-eqn. for differential equation for the temporal evolution of δ:

$$\frac{\mathrm{d}^2\delta}{\mathrm{d}a^2} + \frac{1}{a}\left(3 + \frac{\mathrm{d}\ln H}{\mathrm{d}\ln a}\right)\frac{\mathrm{d}\delta}{\mathrm{d}a} = \frac{3\Omega_M(a)}{2a^2}\delta \tag{7}$$

- growth function $D_+(a) \equiv \delta(a)/\delta(a=1)$ (growing mode)
  - position and time dependence separated: $\delta(\vec{x}, a) = D_+(a)\delta_0(\vec{x})$
- for standard gravity, the growth function is determined by $H(a)$

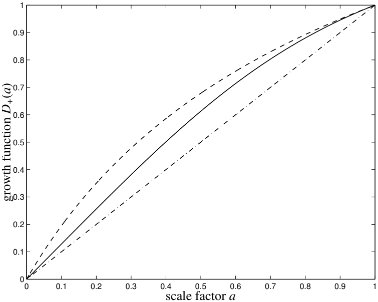
terms in the growth equation



source (thin line) and dissipation (thick line)

- two terms in growth equation:
  - source $Q(a) = \Omega_m(a)$: large $\Omega_m(a)$ make the grav. fields strong
  - dissipation $S(a) = 3 + \mathrm{d}\ln H/\mathrm{d}\ln a$: structures grow if their dynamical time scale is smaller than the Hubble time scale $1/H(a)$

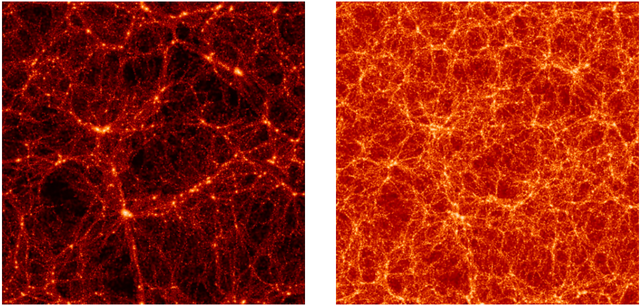
growth function



$D_+(a)$ **for** $\Omega_m = 1$ **(dash-dotted), for** $\Omega_\Lambda = 0.7$ **(solid) and** $\Omega_k = 0.7$ **(dashed)**

- density field grows $\propto a$ in $\Omega_m = 1$ universes, faster if $w < 0$

nonlinear density fields

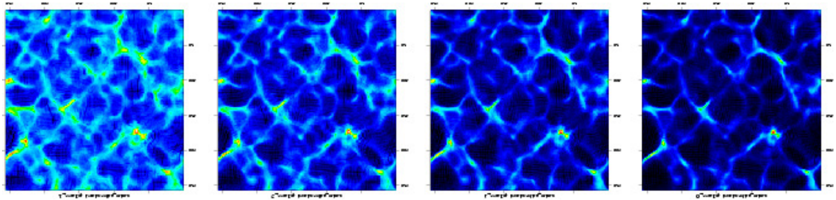


ΛCDM SCDM ($\Omega_m = 1$)

source: Virgo consortium

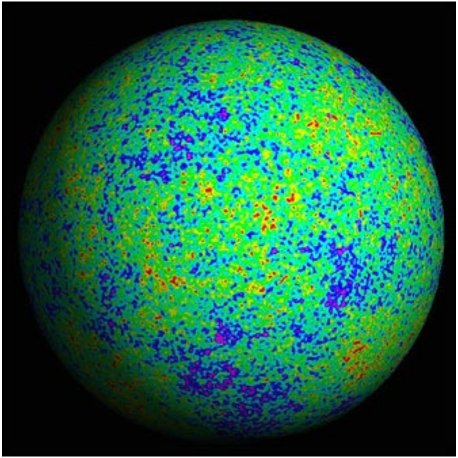
- dark energy influences nonlinear structure formation
- how does nonlinear structure formation change the statistics of the density field?

sequence of structure formation



time sequence of structure formation

inflationary fluctuations in the CMB



source: WMAP

probability density for 1 point

- density of matter is the result of a random experiment
- quote probability density $p(\delta)\mathrm{d}\delta$ for measuring $\delta(\vec{x})$ at a random position $\vec{x}$
- we find that this probability density is Gaussian:

$$p(\delta)\mathrm{d}\delta = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\delta^2}{2\sigma^2}\right)\mathrm{d}\delta$$

  with variance $\sigma^2 = \langle\delta^2\rangle$
- variance can be determined with a spatial integration

probability density for 2 points

- what about a second position? does it have an independent probability $\delta_2 = \delta(\vec{x}_2)$?
- no! the probability for finding δ_2 at $\vec{x}_2$ depends on the value δ_1 at $\vec{x}_1$
- we need a **joint** random process
- formulate this joint process as a Gaussian probability density for 2 variables:

$$p(\delta_1, \delta_2)\mathrm{d}\delta_1\delta_2 = \frac{1}{\sqrt{(2\pi)^2\det(C)}}\exp\left(-\frac{1}{2}\sum_{ij}\delta_i C^{-1}_{ij}\delta_j\right)$$

- with the **covariance matrix** C:

$$C = \begin{pmatrix} \langle\delta_1^2\rangle & \langle\delta_1\delta_2\rangle \\ \langle\delta_1\delta_2\rangle & \langle\delta_2^2\rangle \end{pmatrix}$$

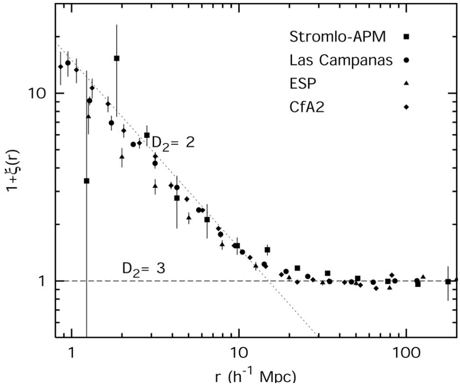
correlation functions

- if the distribution of matter has homogeneous statistical properties, $\langle\delta_1^2\rangle = \langle\delta_2^2\rangle$
- define **correlation function** $\xi(\vec{x}_1, \vec{x}_2) = \langle\delta_1\delta_2\rangle$
- if the field is homogeneous again, the correlation function only depends on the distance $r = \left|\vec{x}_1 - \vec{x}_2\right|$
- rewrite covariance matrix:

$$C = \begin{pmatrix} \langle\delta^2\rangle & \xi(r) \\ \xi(r) & \langle\delta^2\rangle \end{pmatrix}$$

- if $\xi(r) = 0$, the 2 points have independent amplitudes and the Gaussian separates
- if $\xi(r) \neq 1$, the amplitude at $\vec{x}_1$ is **not** independent from the amplitude at $\vec{x}_2$, and the field is **less** random

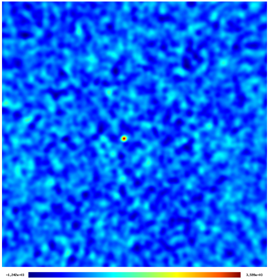
examples of correlation functions



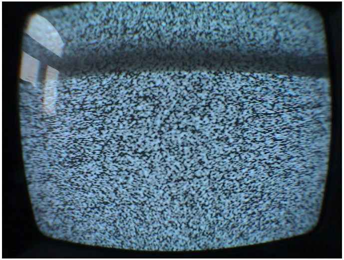
galaxy correlation function, source: B. Jones

- correlation function is a declining power law with slope 2, $\xi(r) \propto r^{-2}$

statistics: correlation function and spectrum



finite correlation length



zero correlation length

correlation function

quantification of fluctuations: correlation function
$\xi(\vec{r}) = \langle\delta(\vec{x}_1)\delta(\vec{x}_2)\rangle$, $\vec{r} = \vec{x}_2 - \vec{x}_1$ for Gaussian, homogeneous fluctuations, $\xi(\vec{r}) = \xi(r)$ for isotropic fields

statistics: correlation function and spectrum

- Fourier transform of the density field

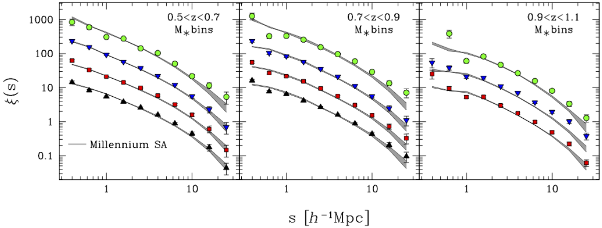
$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\,\delta(\vec{k})\exp(i\vec{k}\vec{x}) \leftrightarrow \delta(\vec{k}) = \int d^3x\,\delta(\vec{x})\exp(-i\vec{k}\vec{x}) \quad (8)$$

- variance $\langle\delta(\vec{k}_1)\delta^*(\vec{k}_2)\rangle$: use homogeneity $\vec{x}_2 = \vec{x}_1 + \vec{r}$ and $d^3x_2 = d^3r$

$$\langle\delta(\vec{k}_1)\delta^*(\vec{k}_2)\rangle = \int d^3r\,\langle\delta(\vec{x}_1)\delta(\vec{x}_1+\vec{r})\rangle\exp(-i\vec{k}_2\vec{r})(2\pi)^3\delta_D(\vec{k}_1-\vec{k}_2) \quad (9)$$

  - definition spectrum $P(\vec{k}) = \int d^3r\,\langle\delta(\vec{x}_1)\delta(\vec{x}_1+\vec{r})\rangle\exp(-i\vec{k}\vec{r})$
  - spectrum $P(\vec{k})$ is the Fourier transform of the correlation function $\xi(\vec{r})$
  - homogeneous fields: Fourier modes are mutually uncorrelated
  - isotropic fields: $P(\vec{k}) = P(k)$

examples of correlation functions



galaxy correlation function, source: VIPER-survey

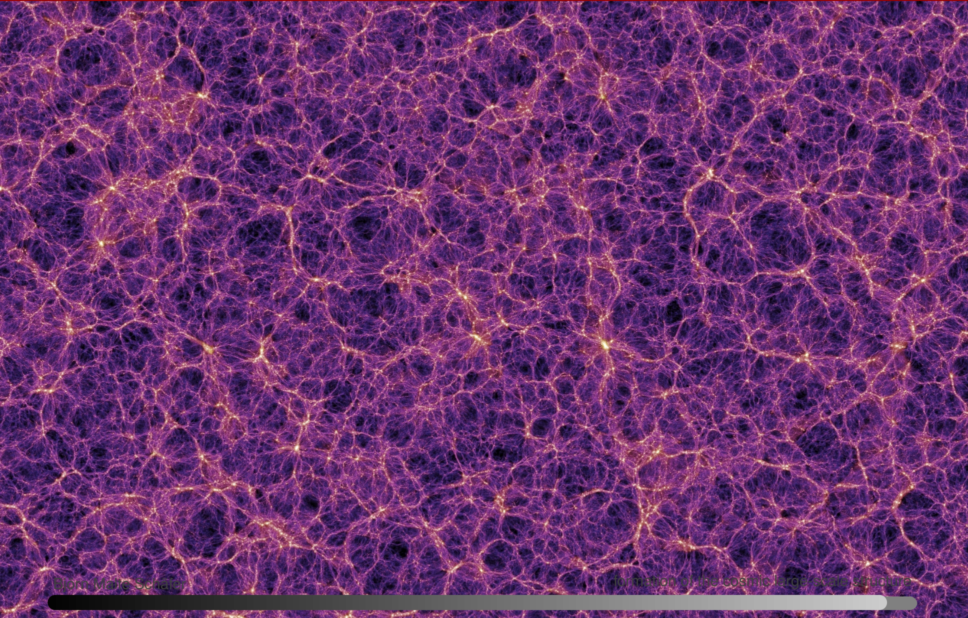
- very good agreement of measured correlation functions with dark matter simulations and galaxy formation models

why correlation functions?

a proof for climate change and global warming

please be careful: we measure the correlation function because it characterises the random process generating a realisation of the density field, not because there is a badly understood mechanism relating amplitudes at different points!
(PS: don't extrapolate to 2014!)

the cosmic web (Millenium simulation)



summary

- inflation generates seed fluctuations in the (dark) matter distribution
- fluctuations form a random field
- description with correlation function $\xi(r)$
- structures grow by self-gravity