cosmic microwave background physics

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1 thermal history

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3 summary
the Universe is filled with radiation corresponding to a temperature of 2.726K

small fluctuations of the temperature of the sky of order $10^{-5}$

radiation from the formation of the first atoms
thermal history of the universe

- temperature of fluids drop while universe expands
- 2 important stages
  1. temperature is high enough to allow nuclear reactions
     → big bang nucleosynthesis \((z \approx 10^{10})\)
  2. temperature is high enough to ionise hydrogen
     → cosmic microwave background \((z \approx 10^3)\)
thermal history of the universe: overview

source: Addison-Wesley
thermal history of the universe: particle interactions

source: particle data group
temperature and Hubble expansion

- Hubble expansion is an **adiabatic** process $\delta Q = 0$
- adiabatic equation: $V^{k-1}T = \text{const}$ with adiabatic index $\kappa \equiv c_p/c_V$
- early times: universe is filled with photons $\kappa = 4/3$ (relativistic gas)
  \[
  T \propto V^{-1/3} \propto a^{-1}
  \]  \hspace{1cm} (1)
- late times: universe is filled with (dark) matter $\kappa = 5/3$ (classical gas)
  \[
  T \propto V^{-2/3} \propto a^{-2}
  \]  \hspace{1cm} (2)
Planck-spectrum for photons

- photons in thermodynamic equilibrium are characterised by the Planck-spectrum

\[ n(p, T) = \frac{g}{(2\pi\hbar)^3} \int_0^\infty dp \frac{4\pi p^2}{\exp(\epsilon(p)/k_B T) - 1} \] (3)

- Planck-spectrum depends only on temperature

- from the number density \( n(p, T) \) of photons we can compute number, energy and pressure by integration

\[ n_\gamma = \frac{g_\gamma \zeta(3)}{\pi^2} \left( \frac{k_B T}{\hbar c} \right)^3, \quad u_\gamma = \frac{g_\gamma \pi^2}{30} \left( \frac{k_B T}{\hbar c} \right)^4, \quad p_\gamma = u_\gamma / 3 \] (4)

- there are two polarisation states, \( g_\gamma = 2 \)

- pure magic: \( u_\gamma \propto a^{-4} \) (dilution and redshift), and at the same time: \( u \propto T^4 \), so \( T \propto a^{-1} \) as predicted from the adiabatic equation
first atoms form

- at high temperatures, the reaction $p + e^- \leftrightarrow H + \gamma$ proceeds in both directions
- as the Universe expands, the temperature drops because of adiabatic cooling
- at low temperatures, the reaction only proceeds in the $\to$-direction and atoms form
- this happens at $\sim 10^4 K$ roughly 300,000 years after the big bang

source: science kids
thermal history

while the Universe is hot, all atoms are ionised: photons scatter off electrons and can’t propagate

Universe cools and atoms form: photons can travel freely and the Universe becomes transparent

we see this radiation redshifted by 1000 today as the microwave background

source: Ned Wright
formation of atoms

- fraction of neutral atoms is a **steep** function of temperature
- while the Universe cools down, the atoms form **really fast**

**source:** wikipedia
cosmic microwave background

- atoms were produced in thermal equilibrium
- photons should follow a Planck-distribution
- redshifted by 1000 since then, from optical to microwave

source: FIRAS@COBE
COsmic Background Explorer

- COBE-satellite

source: NASA
Wilkinson Microwave Anisotropy Probe

source: NASA

- WMAP-satellite
Planck-surveyor

- Planck-satellite

source: ESA
the most important structure on the microwave sky is a dipole

CMB dipole is interpreted as a relative motion of the earth

CMB dispole has an amplitude of $10^{-3}K$, and the peculiar velocity is $\beta = 371 \text{km/s/c}$

$$T(\theta) = T_0 (1 + \beta \cos \theta)$$  

(5)
• the temperature of the sky is not constant, but there are very small fluctuations

• the hot baryon plasma feels fluctuations in the distribution of (dark) matter by gravity

• at the point of (re)combination:
  • hydrogen atoms are formed
  • photons can propagate freely

• perturbations can be observed by two effects:
  • plasma was not at rest, but flowing towards a potential well → Doppler-shift in photon temperature, depending to direction of motion
  • plasma was residing in a potential well → gravitational redshift
subtraction of motion dipole

source: PLANCK
subtraction of Milky Way emission

source: PLANCK

what...
...about those spots everywhere!??
CMB angular spectrum

• analysis of fluctuations on a sphere: decomposition in $Y_{\ell m}$

$$T(\theta) = \sum_{\ell} \sum_{m} T_{\ell m} Y_{\ell m}(\theta) \leftrightarrow T_{\ell m} = \int \, d\Omega \, T(\theta) Y_{\ell m}^*(\theta) \quad (6)$$

• spherical harmonics are an orthonormal basis system

• average fluctuation variance on a scale $\ell \simeq \pi/\theta$

$$C(\ell) = \langle |T_{\ell m}|^2 \rangle \quad (7)$$

• averaging $\langle \ldots \rangle$ is a hypothetical ensemble average. In reality, one computes an estimate of the variance,

$$C(\ell) \simeq \frac{1}{2\ell + 1} \sum_{m=-\ell}^{m=+\ell} |T_{\ell m}|^2 \quad (8)$$
what about those spots?

- we compute the Fourier transformation and measure the angular size of the object (aka the wavelength)
- there’s a peak in the spectrum at 2 degree: that’s the size of the spots
sound waves in the plasma

- processes in the early universe excite sound waves
- we see a superpositions of them in the cosmic microwave background
- there are temperature variations because the plasma is moving around in the sound wave
standard ruler principle

- estimate the **distance** to an object by measuring the **angle** under which it appears
- need to know the true physical size of the object
standard ruler principle

trinity nuclear test, 16 milli-seconds after explosion

- physical size: combine
  1. time since explosion
  2. velocity of fireball

- distance: combine
  1. physical size
  2. angular size
formation of baryon acoustic oscillations

- from a pointlike perturbation, a spherical wave travels in the photon-baryon-plasma
- propagation stops when atoms form
hot and cold patches of the CMB have a typical physical size, related to the horizon size at the time of formation of hydrogen atoms

idea: physical size and apparent angle are related, redshift of decoupling known
standard ruler: measurement principle

- curvature can be well constrained
- assumption: galaxy bias understood, nonlinear structure formation not too important
distance measures: comoving distance

- comoving distance $\chi$ is the distance on a spatial hypersurface between the world lines of a source and the observer moving with the Hubble flow
- photon geodesics are defined by $d\sigma = 0$ (Fermat's principle)
- therefore $c\,dt = -ad\chi$ (from metric), $d\chi = -c\,da/(a^2H)$

$$\chi = c \int_{a_e}^{a_a} \frac{da}{a^2H(a)}$$

- complete analogy to conformal time $d\eta = da/(a^2H)$, such that $\chi = c\eta$
distance measures: angular diameter distance

- Angular diameter distance $d$ is the distance inferred from the angle under which a physical object appears.

- Physical cross section $\Delta A$, solid angle $\Delta \Omega$:
  \[
  \frac{\Delta A}{4\pi a_e^2 \chi} = \frac{\Delta \Omega}{4\pi} \quad \text{(10)}
  \]

- Define $d$:
  \[
  d \equiv \sqrt{\frac{\Delta A}{\Delta \Omega}} = a_e \chi \quad \text{(11)}
  \]
relation between distance and redshift

cosmological distances vs. redshift $z$
parameter sensitivity of the CMB spectrum

source: Wayne Hu
CMB simulator

http://www.strudel.org.uk/planck/

Planck paper model

http://planck.cf.ac.uk/news/make-your-own-planck-model
baryon acoustic oscillations in the galaxies

- baryon acoustic oscillations: the (pair) density of galaxies is enhanced at a separation of about 100\text{Mpc}/h comoving
- idea: angle under which this scale is viewed depends on redshift
• we can today observe the radiation from the formation of atoms
• the atoms formed at a temperature of 3000K at a redshift of 1000, and today
  1. temperature is 3K
  2. frequency is 160GHz
  3. wave length is 3mm
• the optical light is shifted to microwaves by cosmological redshifting
• redshifting corresponds to adiabatic cool-down in the expansion
summary: spots in the CMB

- the temperature has tiny fluctuations: there are spots in the CMB
- sound waves are excited in the plasma in the early Universe
- the sound waves travel until atoms form
- a standard ruler of size $c_s \times \Delta t$ is established
- we observe this standard ruler under an angle of $1 \ldots 2$ degrees
- we know how far the cosmic microwave background is away, and have an integrated measure of the Hubble function