Exercises, Day 2

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Names: ____

Spherical coordinates

Today's exercise will consist of a worked-out example for the new concepts introduced in yesterday's and today's lecture.

Consider the usual spherical coordinates defined by

$$\begin{aligned} x &= r \cdot \sin\theta \cdot \cos\phi \\ y &= r \cdot \sin\theta \cdot \sin\phi \\ z &= r \cdot \cos\theta \end{aligned}$$

(a) Compute the inverse transformation $r(x, y, z)$, $\theta(x, y, z)$, $\phi(x, y, z)$.

(b) From the Euclidean metric for the Cartesian coordinates x, y, z, write down the line element in spherical coordinates. Which are the components of the metric?

(c) Using the transformation formula for vectors, compute the expression (that is, the components v^i) for the vector fields

$$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \quad \text{and} \quad x \cdot \frac{\partial}{\partial z}$$

in these coordinates.

(d) What are the derivatives ∂_r, ∂_ϕ, ∂_θ of these vector fields?

(e) Compute the non-zero connection coefficients Γ^i_{jk}.

(f) What are the covariant derivatives for the vector fields defined in (c)?

(g) Consider the straight line $y = 2 \cdot x$. Re-write in spherical coordinates and use the geodetic equation in polar coordinates to show that it is indeed a geodetic line.