Kosmologie

Lehrerfortbildung allgemeine Relativitätstheorie

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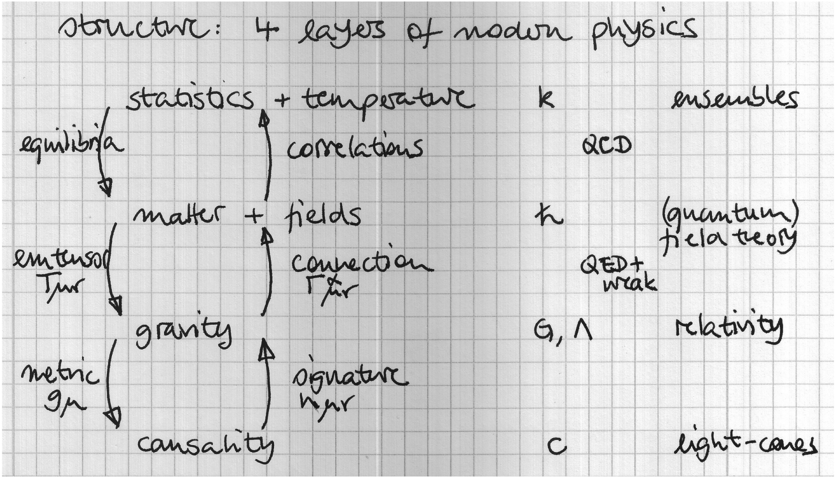
modern cosmology

structure of modern physics?



structure: 4 layers of modern physics

statistics + temperature k ensembles

equilibria correlations QCD

matter + fields $\hbar$ (quantum) field theory

em tensor $T_{\mu\nu}$ connection $\Gamma^{\alpha}_{\mu\nu}$ QED + weak

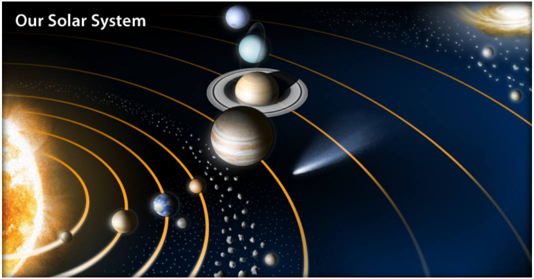
gravity G, Λ relativity

metric $g_{\mu\nu}$ signature $\eta_{\mu\nu}$

causality c light-cones

structure of modern physics

Newton: geometry and motion



solar system, source: NASA

- why do the planets move and what keeps their orbits stable?
- Greek philosophers had funny ideas for this: the planets could be fixed on crystal spheres, and the radii of these spheres were in harmonic proportions to each other
- but there must be a better explanation, based on gravity as a force

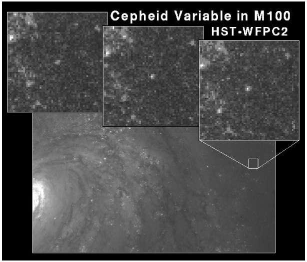
Newton: geometry and motion

- gravity was well known, but Newton was the first one who thought that the earth's gravity extends to the moon
- additionally, he had a law of motion worked out, $m_i\ddot{x} = -m_s\nabla\Phi$
- with that, he could solve Kepler's problem and find out that $\Phi \propto 1/r$
- Newton's constant of gravity

  $$G \sim 10^{-11}\frac{m^3}{kg \times s^2} \tag{1}$$

  where Kepler's law is hidden: distance^3=orbital time^2
- with Kepler's law one could infer the distances of all planets from their orbital periods, because the gravitational constant and the mass of the sun was known, from the orbit of the earth
- but the geometry was still Euclidean

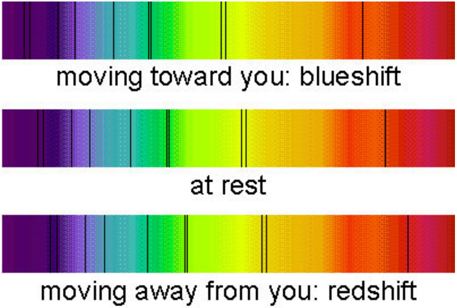
Leavitt: what's the distance to galaxies?



Cepheid variable star, source: Hubble space telescope

- Cepheid stars are a special type of variable stars, they change their brightness periodically
- brightness-period relation can be investigated with local stars where parallax measurements have been done
- period → true brightness, apparent brightness → distance
- Magellanic clouds (satellite galaxies of the Milky Way) are far away!

Hubble: galaxies are moving away from us?



Doppler shift on spectral lines, source: supernova acceleration probe

- spectral lines of distant galaxies is shifted towards the red part of the spectrum
- move the galaxies away from us, and is this analogous like the sound of an ambulance?
- (in reality, the explanation is much, much, much better!)

Hubble expansion

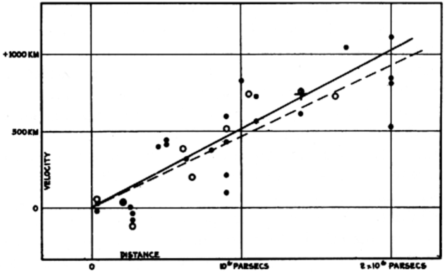


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

distance-redshift relation of galaxies, source: Hubble's original work

- Hubble measured distances and redshifts of galaxies
- distant galaxies **seem** to move away from us, with a velocity proportional to distance, the universe is **not static**: $\upsilon = Hr$
- a distance of 1000 km becomes longer by 7 cm in 1000 years

Newtonian cosmology

- use Newtonian gravity for describing dynamics of the universe
- basic assumptions
  1. Euclidean (flat) space
  2. homogeneous distribution of matter
  3. isotropic expansion
- consider a test particle on the surface of a sphere

$$\ddot{r} = -\frac{GM}{r^2} \quad \text{with} \quad M = \frac{4\pi}{3}\rho r^3 \tag{2}$$

- results from Newton's law

$$\ddot{r} = -\frac{\partial}{\partial r}\Phi \quad \text{with} \quad \Delta\Phi = 4\pi G\rho \tag{3}$$

- comoving coordinate: $r = ax$, a: scale-factor

$$\ddot{a} = -\frac{4\pi G}{3}\rho a \quad \rightarrow \quad \frac{\dot{a}^2}{2} = \frac{GM}{a} + E \tag{4}$$

critical density

- E: integration constant, 3 possible types of solutions
  1. $E > 0$: elliptic
  2. $E < 0$: hyperbolic
  3. $E = 0$: parabolic
- evolution of scale-factor a:

$$E = \frac{\dot{a}^2}{2} - \frac{GM}{a} \quad \rightarrow \quad \frac{E}{a^2} = \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 - \frac{GM}{a^3} = \frac{H^2}{2} - \frac{4\pi G}{3} \tag{5}$$

- conditions for parabolic solution:

$$E = 0 \leftrightarrow \rho_{\mathrm{crit}} = \frac{3H_0^2}{8\pi G} \tag{6}$$

- observationally: $\rho_{\mathrm{obs}} = \Omega_m \rho_{\mathrm{crit}}$ with $\Omega_m = 0.25$

supernova 1994D in the galaxy NGC 4526

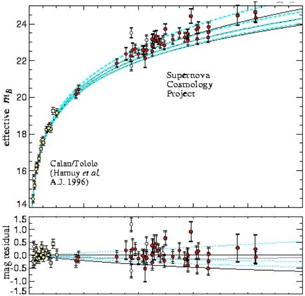
a bright supernova in its host galaxy, source: Hubble space telescope

- go to **much larger** distances than Cepheid stars: **supernovæ**

what’s so special about supernovæ?

- we can see supernovæ at really large distances up to 10^{25} meters
- because they always happen if the star reaches a certain mass (the Chandrasekhar mass, about $1.4M_{\odot}$), they’re almost equally bright
- they’re comparatively easy to spot
- the bright phase lasts about a month
- we can try to measure the distance and check out the relation to the expansion velocity
- they’re **not rare** events: there’s a supernova every 100 years in our own Milky Way

distance-redshift relation



distance redshift relation of supernovæ, source: supernova cosmology project

- combine two measurements:
  1. from the brightness of the supernova one can estimate the distance
  2. from the redshift of the galaxy one can estimate the velocity
- we can measure the relation between velocity and distance, and see if the velocity is constant or if it changes

distance-redshift relation

inflation of a balloon, source: science blogs

- inflating a balloon: very problematic analogy
  1. we see an **apparent velocity** of the supernova away from us
  2. we as observers and the supernova **stay at their coordinate**
- metric changes and we relate the change in scale factor to the integrated distance from us to the supernova

Einstein: geometry, dynamics and gravity

- Newton's idea for gravity as a force that keeps the planets bound to the Sun uses Euclidean geometry
- gravity decreases like distance^2, very much like the electrostatic Coulomb-force
- but there are new discoveries: gravity is fundamentally different compared to electrostatics
  - space and time are combined into a spacetime
  - gravity changes the properties of this to be **non-Euclidean**
  - geometry can even change from point to point
- Einstein formulated general relativity as a theory of gravity based on differential geometry, a new type of varying non-Euclidean geometry
- he achieved something the Greek mathematicians would have liked: a unification of geometry, dynamics and gravity

general relativity: geometry + gravity + dynamics

- split up coordinate and distances between coordinate points
- **geometry**: distance measurement, which can vary locally: **metric**

  $$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right]$$

  as a generalisation to $\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$
- **dynamics**: particle motion minimises s: **geodesic**

  $$\delta\int \mathrm{d}s = 0$$

- **gravity**: gravitational fields Φ change the geometry: **curvature**

  $$\mathrm{d}s^2 = c^2(1+2\Phi)\mathrm{d}t^2 - (1-2\Phi)\left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right]$$

  at least for weak gravitational fields, strong fields are more complicated ;-)
- look at gravitational constant $G/c^2 \sim 10^{-28} m/kg$

cosmology based on general relativity?

- we think that the metric of the universe changes according to

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - a^2(t)\left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right] \tag{7}$$

  with the scale factor a
- the dynamics of $a(t)$ depends on everything that can generate a gravitational field
- redshift $z = \Delta\lambda/\lambda$ of a spectral line is not a motion redshift, but $1 + z = 1/a$: photons are measured with a different metric at different times
- $a(t)$ is increasing with time, $\dot{a} > 0$
- there was an instant, where $a = 0$: that's the big bang, at which the measurement of distances was not possible
- right now, the $\ddot{a} > 0$, meaning that the expansion rate increases, under the influence of **dark energy**: in fact, gravity becomes **repulsive** on large scales and the universe falls apart

field equation

- Einstein's field equation relates the geometry $g_{\mu\nu}(x)$ to the energy-momentum content $T_{\mu\nu}$ (with $T_{tt} = \rho c^2$) in a local differential equation
$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{8}$$
- the tt-part reminds of the Poisson-equation for Newtonian-gravity
$$\Delta\Phi = 4\pi G\rho \quad \rightarrow \quad \Delta\frac{\Phi}{c^2} = \frac{4\pi G}{c^4}(\rho c^2)$$
which explains the prefactor
- the field equation is of second order combining derivatives in space and in time, and generalises the Poisson equation

regimes of relativity

- relativity links geometry to matter in a second-order differential equation
- relativity does **not** make a difference between time t and space r
- identify following typical regimes:
  1. weak, static fields: solar system, motion of planets (except Mercury)
     $$\mathrm{d}s^2 = c^2(1+2\Phi)\mathrm{d}t^2 - (1-2\Phi)\left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right]$$
  2. strong, static fields: Schwarzschild metric around black holes
     $$\mathrm{d}s^2 = c^2\left(1 - \frac{2GM}{c^2 r}\right)\mathrm{d}t^2 - \left(1 + \frac{2GM}{c^2 r}\right)^{-1}\left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right]$$
  3. weak, time-evolving fields: gravitational waves
     $$\Box h_{\mu\nu} = 0$$
  4. strong, time-evolving fields: cosmology
     $$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - a^2(t)\left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right]$$

solving the field equation

- solve the field equation with **homogeneity + isotropy assumption** for an **ideal, dissipationless, stable fluid**: FLRW-cosmology
- Markus explained: metric $g_{\mu\nu}$ → Christoffel-symbols $\Gamma^{\alpha}_{\mu\nu}$ → curvature $R^{\alpha}_{\mu\beta\nu}$ → Ricci-tensor $R_{\mu\nu}$ → Ricci-scalar R
- end up at **two** Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^3} = -4\pi G p$$

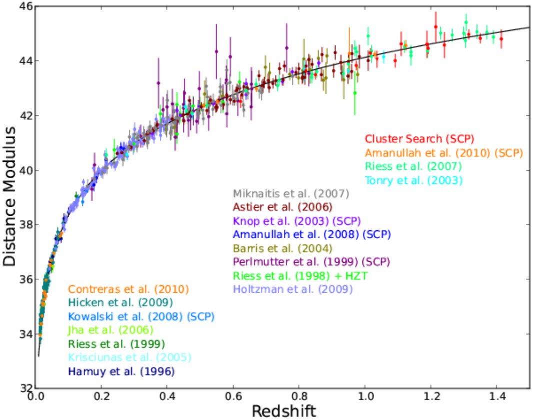
- evolution of the scale factor a depending on the density ρ and pressure p of the cosmological fluids
- second order
- characterise ρ and p of every cosmological fluid

why is relativity necessary

- in fact, many results would follow from a Newtonian description
- **Copernican principle**:
  large-scale isotropy and homogeneity, other observers would see the same expansion from their point of view
- **negative pressure**:
  accelerated expansion due to dark energy or the cosmological constant
- **effects of global curvature**:
  the actual cosmological model has only small amounts of curvature
- **critical density**:
  density of matter is very close to that value

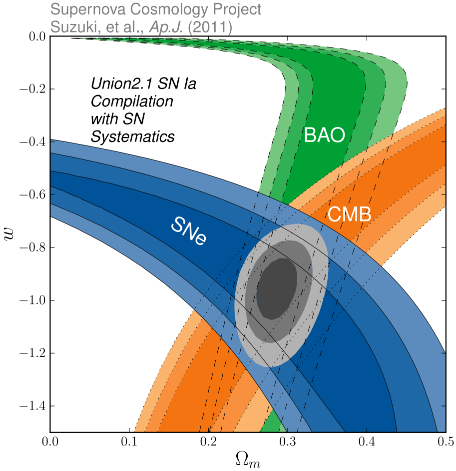
funny consequences: empty space can't be static and Euclidean

distance-redshift relation of supernovae



distance redshift relation of supernovæ, source: supernova cosmology project

p and ρ from supernovae



pressure and density from supernovae, source: supernova cosmology project

what's the relation to modern physics?

- perhaps one should rather imagine, that the universe shows us physical processes on different scales
  1. $a = 10^{-24}$ cosmic inflation ends
  2. $a = 10^{-14}$ baryogenesis, QCD phase transition
  3. $a = 10^{-13}$ electroweak symmetry breaking
  4. $a = 10^{-10}$ formation of light atomic nuclei
  5. $a = 10^{-3}$ formation of atoms
  6. $a = 0.01$ first stars
  7. $a = 0.1$ first galaxies

  until $a = 1$ today
- because the temperature of photons scales like $T \propto a^{-1}$, we see equilibrium processes on the various scales
- if one includes the expansion since the big bang, there are ~ 60 orders of magnitude in scale, which provide the arena of modern physics: from the Planck length 10^{-35}m up to the Hubble length 10^{+25}m